Application of GRG Technique to Estimate Reservoir Capacity

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Abstract: **One of the most important physical characteristic of reservoirs is their area capacity curves. Area capacity curves are required for determination of water surface area, reservoir capacity, outlet sill level, reservoir sediment distribution. Several semi-empirical and empirical equations have been proposed to predict the area capacity curves precisely. These semi-empirical and empirical approaches generally employ graphical method to obtain the reservoir coefficients. Use of Generalized Reduced Gradient (GRG) technique, a powerful optimization tool, has been proposed in this paper to estimate the reservoir coefficients precisely. A comparative analysis of trial-error based graphical method and optimization approach shows that GRG technique is more reliable tool for estimation of reservoir coefficients accurately. The reservoir capacities calculated from optimization approach were found to be more accurate than the graphical method.**

Keywords: **Area capacity curve; GRG; Optimization; Reservoir; Trial-error.**

1. INTRODUCTION

Dams are constructed across the river for hydropower generation, flood control, irrigation and water supply. One of the most important physical characteristic of dams and reservoirs are their area capacity curves. Area capacity curves are obtained by planimetering the area enclosed within each contour line of the reservoir area. Reservoir area capacity curves are of paramount importance for design engineer as they are the source of useful information such as water surface area at various elevation, reservoir capacity, reservoir sediment distribution and reservoir classification. Reservoir surveys are made time to time to calculate the amount of sediment deposit by calculating the difference in original capacity and the capacity from recent capacity elevation curve. Further, area capacity curves are required for estimating useful life of reservoir, dead storage at the dam site, outlet sill elevation, level of penstock for efficient functioning and predicting water level in case of backwater conditions. Owing to its wide importance obtaining reliable area-capacity equations deserves a special place in water resource engineering.

Reservoir capacity equation is generally expressed as parabolic function of reservoir depth above the streambed represented by

$$V_y = a + by + cy^2 \tag{1}$$

Where V_y is the capacity of reservoir at depth y; y is the depth of water above the reservoir bed; a, b and c are the coefficients.

The three coefficients in the above equation can be determined by using ACAP, a computer programme as used by U. S. Bureau of Reclamation. In this programme the area capacity is fitted either by using least square set of equations or cubic spline. Mohammadzadeh-Habili et al. [1] utilized the similarity between natural logarithmic function curve and reservoir capacity curve to drive dimensionless capacity curve equation with only one unknown coefficient known as reservoir coefficient. They also proposed a relation between reservoir coefficient and reservoir shape factor. Kaveh et al. [2] too proposed a dimensionless capacity curve equation with single unknown coefficient which could easily be determined using graphical method. Kaveh et al. [2] also compared their results to the equation proposed by Mohammadzadeh-Habili et al. considering data sets of twenty reservoirs and concluded that the new proposed equations are more precise. Although the equations proposed by Kaveh et al. [2] were simple but the accuracy of the proposed method can be increased by using appropriate optimization technique rather than trial-error based graphical method. Therefore, the present work proposes the use of simple spreadsheet based non-linear optimization approach to obtain the reservoir coefficient using equations as proposed by Kaveh et al. [2].

The dimensionless reservoir capacity equation proposed by Kaveh et al. is presented as

$$p = v^{N/2} \tag{2}$$

Where p is the relative depth defined as

$$p = \frac{y}{y_m} \tag{3}$$

and v is the relative volume defined as

$$v = \frac{V_y}{V_m} \tag{4}$$

V_m is the reservoir capacity at maximum pool level; y_m is the maximum water depth in the reservoir; N is the reservoir coefficient

On rearranging equation (2) may be written as

$$v = p^{2/N} \quad (5)$$

Similarly, dimensionless reservoir area equation proposed by Kaveh et al. is represented as

$$a = p^{(2-N)/N} \quad (6)$$

Where a is the relative area defined as

$$a = \frac{A_y}{A_m} \quad (7)$$

A_y is the area of reservoir at depth y; A_m is the reservoir area at maximum pool level.

Analytical equation to estimate the reservoir coefficient was also proposed which may be presented as

$$N = \frac{2V_m}{y_m A_m} \quad (8)$$

Data

Reservoir elevation-area-capacity data of ten randomly selected reservoirs as published by United State Bureau of Reclamation (USBR) [3-13] has been used to evaluate the performance of GRG technique. Some of the important physical characteristics of these reservoirs is tabulated below.

2. METHODOLOGY

Most of the earlier studies employed graphical method to obtain the reservoir coefficients, present study is a novel attempt of employing a simple spreadsheet based nonlinear optimization approach to obtain the reservoir coefficients. The observed reservoir area and capacity data at various elevations for all the reservoirs was fed into the excel spreadsheet. The maximum depth (y_m) was calculated as the difference between the elevation corresponding to largest volume or area and elevation corresponding to reservoir bed. The depth, volume and area were converted into relative depth, relative volume and relative area using equations (3), (4) and (7) respectively. Thus, the data sets were converted into dimensionless form with entries ranging from zero to unity. In order to use the GRG technique, the relative depth-area and relative depth-volume relationship were modelled using spreadsheet. The relative area and relative volume of the reservoir was then estimated on the basis of assumed value of reservoir coefficient 'N' using equation (5) and equation (6). To obtain the optimal value of reservoir coefficient 'N' corresponding to relative area using GRG technique,sum of square of deviation between observed relative area and estimated relative area was set to minimization defined as

$$\text{Min SSE} = \sum_{i=1}^{N}[a_{o_i} - a_{e_i}]^2 \quad (9)$$

Where a_o is observed relative area and a_e is the estimated relative area corresponding to any relative depth p_i.

Similarly, to obtain the optimal value of reservoir coefficient 'N' corresponding to relative volume sum of square of deviation between observed relative volume and estimated relative volume was set to minimization defined as

$$\text{Min SSE} = \sum_{i=1}^{N}[v_{o_i} - v_{e_i}]^2 \quad (10)$$

Table 1 Important physical properties of selected reservoirs

Serial Number	Name of Reservoir	Maximum Depth (y_m) (feet)	Maximum Area (A_m) (acre)	Maximum Volume (V_m) (acre-feet)
1	Boysen	137	30894	1491924
2	Box Butte	61	2116	47797
3	Cascade	74	32967	1059857
4	Cedar Bluff	128	16510	730636
5	Clark Canyon	122.9	6606.2	329430
6	Millerton	297.6	5110	555500
7	Nambe Falls	143	74.4	2913
8	Pueblo	200	8027	535807
9	Pishkun	84.5	1741	54852
10	UTE	122	11237.1	397996

Where v_o is observed relative volume and v_e is the estimated relative volume corresponding to any relative depth p_i.

From the observed data analytical value of reservoir coefficient 'N' was also calculated using the equation (8).

To assess the accuracy of estimated area and capacity their absolute values were estimated using the equations (11) and (12) respectively and compared with the observed data using equation (13)

$$A = A_m p^{(2-N)/N} \quad (11)$$

$$V = V_m p^{2/N} \quad (12)$$

$$\text{Root Mean Square (RMSE)} = \sqrt{\frac{\sum (X-Y)^2}{N}} \quad (13)$$

Xis observed quantity and Y is the estimated quantity.

The classification of reservoir was carried on the basis of recommendation of Borland and Miller given in table 2. Reservoir shape factor is defined as the reciprocal of slope of depth versus capacity curve when plotted on double log scale. The logarithmic form of equation (2) may be represented as.

$$\log p = \log v^{N/2}$$

$$\text{or } \log p = \frac{N}{2} \log v$$

from the above definition shape factor that is reciprocal of the slope of above equation is

$$M = \frac{2}{N} \quad (14)$$

The shape factor was obtained using equation (14) and the classification of reservoir was done on the basis of table (2).

Table 2 Reservoir classification for sediment distribution by Borland and Miller (1958).

M	Reservoir type	Standard classification
3.5-4.5	Lake	I
2.5-3.5	Flood plain – Foothill	II
1.5-2.5	Hill	III
1-1.5	Gorge	IV

3. GRG TECHNIQUE

Lasdon et al. [14] developed Generalized Reduced Gradient (GRG) technique which is basically a nonlinear extension of simplex method. GRG and its specific implementations have been proved in use over many years as one of the most robust and reliable approaches for solving complex nonlinear programming problems (Lasdon and Smith, [15]). GRG solver is capable of solving both constrained as well as well as unconstrained non-linear optimization problems. The search direction of GRG code is dictated either by quasi-Newton method or by conjugate gradient method, depending on the available storage GRG can automatically switch between quasi-Newton and conjugate gradient method. However, the default choice is quasi-Newton method. The quasi-Newton method relies on approximation to Hessian matrix and requires more storage, while the conjugate gradient method does not require much storage. Models for optimization problems can be built using either excel spreadsheet or using custom programs like C or C^{++}. In the present study GRG solver embedded within Microsoft excel was used to obtain the optimum values of the reservoir coefficient.

Microsoft excel is one the most widely used software to solve various problems of engineering and business. Weiss and Gulliver [16] demonstrated the utility of excel for analysing hydraulic design projects. Jewell [17] proposed the application of TK solver for teaching practical problems in hydraulics and emphasized on the importance of equation solvers as an educational tool in mastering hydraulic design within a limited duration. Grabow and McCornick [18] used excel to assess water quality, its allocation and management. Precise calculation of critical depth in open channel was performed by Bhattacharjya [19] using excel solver. Barati [20] applied excel solver for estimating the parameters of nonlinear Muskingum flood routing using benchmark data set of Wilson (1974). Non-linear Muskingum parameters estimated by excel solver resulted in minimum sum of square of error among Least square method (LSM), Hook-Jeeves and Davidson-Fletcher-Powell (HJ+DFP), Genetic algorithm (GA), Immune clonal selection algorithm (ICSA). Muzzamil et al. [21] used GRG solver to establish stage discharge relationship for Lakhwar dam site. Zakwan and Muzzammil [22] applied GRG technique to model the nonlinear form of Muskingum flood routing equation demonstrating that nonlinear form of Muskingum flood routing equation estimates the outflow more accurately. Zakwan et al. [23] used GRG technique to obtain suction head, porosity and Darcy conductivity from infiltration data (Zakwan [24]).

4. RESULTS AND DISCUSSION

The reservoir coefficients 'N' were estimated using the non-linear optimization technique for minimization of error in relative volume and as well as relative area. The reservoir coefficients resulting from minimization from relative area were almost same as those obtained by Kaveh et al. using graphical method but, a significant difference in the reservoir coefficient calculated by minimization of error in relative volume obtained from optimization and graphical method was observed. The resulting sum of square of error in relative volume (SSE_v) for the optimization approach was much lesser than those reported by Kaveh et al. for graphical method. The effect of this difference in the value of reservoir coefficient becomes more pronounced when absolute reservoir capacity was calculated for different values of reservoir coefficient using the equation (12)

Table 3 Performance index of estimated capacity.

Name of Reservoir	SSE_v (equation 10)		% Reduction in SSE_v from present study	RMSE of capacity (Acre-ft)		
	Kaveh	Present study		Kaveh	Analytical method	Present study
Boysen	0.00116	0.00066	42.7	9418	21237	7128
Box Butte	0.00066	0.00182	63.7	425	386	255
Cascade	0.00083	0.00034	64.2	7470	6994	4469
Cedar Bluff	0.00047	0.00011	77.4	2846	1541	1352
Clark Canyon	0.01119	0.00411	63.3	6259	8847	3791
Millerton	0.00119	0.00026	78.5	3284	4039	1521
Nambe Falls	0.00368	0.00053	85.7	32.25	25.44	12.2
Pueblo	0.00133	0.00035	73.9	4071	8884	2081
Pishkun	0.0391	0.00132	66.3	965	786	457
UTE	0.00281	0.00038	86.6	5270	4784	1926

Table 4 Reservoir coefficients obtained by different approaches

Serial Number	Name of Reservoir	Reservoir type	N from equation (8)	N from equation (9)		N from equation (10)	
				Kaveh	Present study	Kaveh	Present study
1	Boysen	II	0.705	0.7077	0.7077	0.7479	0.7345
2	Box Butte	II	0.7406	0.7398	0.7398	0.7004	0.7219
3	Cascade	III	0.8689	0.8689	0.8684	0.8297	0.8502
4	Cedar Bluff	II	0.6915	0.69	0.69	0.6988	0.6884
5	Clark Canyon	II	0.8115	0.8175	0.8176	0.6867	0.7326
6	Millerton	II	0.7306	0.7302	0.7302	0.694	0.7098
7	Nambe Falls	I	0.5476	0.5501	0.5501	0.5038	0.5286
8	Pueblo	II	0.6675	0.6696	0.6696	0.697	0.7176
9	Pishkun	II	0.7457	0.7346	0.7366	0.836	0.7963
10	UTE	II	0.5806	0.5718	0.5718	0.6511	0.6131

The values of Root mean square error (RMSE) in absolute capacity and SSE_v (sum of square of error in relative volume) are shown in table 3. From table 3, it is clear that capacity of reservoir estimated by optimization method are much closer to the observed data as compared to the graphical proposed by Kaveh et al.[2] and analytical method. Also the values of reservoir coefficients resulting from optimization method matches the analytical values of reservoir coefficient more closely as compared to the graphical method. The values of reservoir coefficient and classification of reservoir has been reported in table 4.

5. CONCLUSION

In the present paper the reservoir area-elevation and capacity elevation relations were developed using the nonlinear optimization technique. The values of reservoir coefficient for reservoir area curve resulting from graphical and optimization approach were almost same. However, there was considerable improvement in the estimation of reservoir capacity using the optimization approach. The proposed optimization approach is much easier and saves time consumed in trial and error and hence it can be used as an alternative to graphical method to develop reservoir area-elevation and capacity elevation relations.

Spreadsheet based GRG technique is very simple, quick and even without the knowledge of complicated programming techniques and exact mathematics of optimization it can be used to estimate the parameters incomplex nonlinear relationships quite accurately. GRG technique opens a scope for easy solution of wide variety of problems not only in engineering hydrology but also other fields of engineering.

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