Unsteady MHD Free Convective Flow through Porous Medium over a vertical Plate with Variable Temperature and Concentration

C. Sucharitha[1], S.V.K. Varma[1], C. Madhavareddy[2], K.S. Balamurugan[2]
Department of Mathematics
[1] S.V. University, Tirupati, 517520 (India),
[2] RVR&JC College of Engineering, Guntur, 522019 (India)
Email- muruganbalaks@gmail.com
Received 08 February 2017, received in revised form 22 February 2017, accepted 02 March 2017

Abstract: **The objective of the present study is to investigate the effects of chemical reaction and radiation on unsteady MHD free convection fluid flow embedded in a porous medium with time dependent suction and temperature gradient heat source. The problem has been solved and obtained solutions for velocity, temperature, concentration, skin friction coefficient, rate of heat transfer and rate of mass transfer using perturbation technique in two different cases namely Case (I): Flow past a stationary plate with variable temperature and concentration. Case (II): Flow past a moving plate with variable temperature and concentration. The effects of various parameters such as Prandtl number *Pr,* thermal Grashof number *Gr*, mass Grashof number *Gm,* Schmidt number *Sc*, chemical reaction parameter *Kr,* magnetic field parameter *M,* thermal radiation parameter *Nr* and heat source parameter *Q* are discussed in detail through graphs and tables.**

Keywords: **Unsteady, MHD, free convection, porous medium.**

1. INTRODUCTION

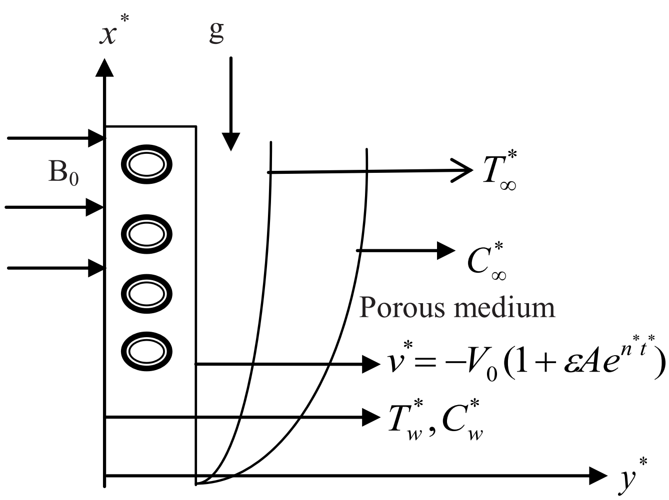
The study of radiative heat and mass transfer in convective flow is found to be most important in scientific and technologies. The applications are often found in situations such as fiber and granules insulations, geothermal systems in cooling and heating of chambers. Further the magneto convection plays an important role in the magnetic separation of molecular semi conducting materials. MHD flows assumes greater significance in several engineering and biological systems when the flow is considered over a permeable boundary. The mass is transferred purely by molecular diffusion resulting from the concentration gradient, if mass transfer takes place in a fluid at rest. A number of investigations have been carried out with combined heat and mass transfer under the supposition of different physical real life situations. Due to the prime importance of heat and mass transfer in chemical reaction, industrial process, and the problem received considerable attention in recent years. In the last few years, many investigations [1- 11], have been carried out with respect to the present work.

Radiation and mass transfer effects on MHD free convective unsteady fluid flow embedded in porous medium with heat generation / absorption was studied by Shanker et al. [12]. Effects of chemical reaction and radiation absorption on unsteady hydromagnetic free convection flow of a incompressible, viscous, electrically conducting fluid with heat and mass transfer past a moving vertical porous plate with time dependent suction and constant mass flux in the presence of heat source in a slip flow regime was studied by Sudharshan Reddy et al. [13]. Balamurugan et al. [14] analyzed unsteady MHD free convective flow past a moving vertical plate with time dependent suction in the presence of heat source in a slip flow regime with slip due to jump in temperature and concentration. Vijaya Kumar et al. [15] investigated the effects of radiation and induced magnetic field on MHD mixed convective chemically reacting fluid over a porous vertical plate. Chemical reaction and radiation effects on unsteady MHD free convective fluid flow embedded in a porous medium with time dependent suction with temperature gradient heat source was studied by Seshaiah et al. [16].

This paper focuses on the effects of thermal radiation, time-dependent suction and chemical reaction on two dimensional MHD free convective fluid flow over a semi-infinite vertical plate, moving exponentially with time, in the occurrence of temperature gradient heat source under the influence of applied transverse magnetic field normal to the flow in two different cases of boundary conditions. **Case (I):** Flow past a stationary plate with variable temperature and concentration. **Case (II):** Flow past a moving plate with variable temperature and concentration. In both cases estimated solutions have been derived for velocity, temperature, concentration profiles, skin friction, rate of heat transfer and rate of mass transfer using perturbation technique. The obtained results are discussed with the help of graphs to detect the effect of various parameters like Schmidt number (*Sc*), Prandtl number (*Pr*), Magnetic field parameter (*M*), thermal radiation parameter (*Nr*), Grashof number for mass transfer (*Gm*), thermal Grashof number (*Gr*) and Chemical reaction parameter (*Kr*), and heat source parameter (*Q*).

2. FORMULATION OF THE PROBLEM

We consider a problem of unsteady two dimensional, laminar boundary layer flow of viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration boundary effects. A variable time-dependent suction velocity $v^* = -V_0(1+\varepsilon A e^{n^* t^*})$ is considered normal to the flow. Where, A is the suction parameter and $\varepsilon A << 1$. Here the minus sign indicates that the suction is towards the plate. The plate is taken along x^*- axis in vertical upward direction against the gravitational field. y^* - axis is taken normal to the flow in the direction of applied transverse magnetic field. Further, due to semi-infinite plane surface assumption the flow variables are the functions of y^* and t^* only. The following assumptions are made in the governing equations. i) The viscous dissipation is neglected. ii) The Joule dissipation is neglected. iii) The induced magnetic field is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. Porous medium



Physical configuration and coordinate system

Under above assumptions and by usual Boussinesq's approximation the unsteady flow is governed by the following set of equations

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\sigma B_o^2}{\rho} u^* + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\upsilon}{k^*} u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = v \frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y^*} + \frac{\overline{Q}}{\rho C_p} \frac{\partial}{\partial y}(T - T_\infty) + R^*(C - C_\infty) \tag{3}$$

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = v \frac{\partial^2 C}{\partial y^{*2}} - k_r'^2 C \tag{4}$$

The corresponding initial and boundary conditions are

Case (I):

$$u^* = 0, \quad T = T_w + \varepsilon(T_w - T_\infty)e^{n^* t^*},$$
$$C = C_w + \varepsilon(C_w - C_\infty)e^{n^* t^*} \; at \quad y^* = 0$$
$$u^* \to 0, \quad T \to T_\infty, C \to C_\infty \; as \quad y^* \to \infty \tag{5}$$

Case (II):

$$u^* = U_0, \quad T = T_w + \varepsilon(T_w - T_\infty)e^{n^* t^*},$$
$$C = C_w + \varepsilon(C_w - C_\infty)e^{n^* t^*} \; at \quad y^* = 0$$
$$u^* \to 0, \quad T \to T_\infty, C \to C_\infty \; as \quad y^* \to \infty \tag{6}$$

By using Rosselant approximation, we can write the radiative flux q_r as:

$$q_r = \frac{4\sigma^*}{3k^*} \frac{\partial T^{*4}}{\partial y^*} \tag{7}$$

It is assumed that the temperature differences within the flow are sufficiently small so that T^{*4} can be expanded in a Taylor series about the free stream temperature T^*_∞ so that after rejecting higher order terms:

$$T^{*4} \approx 4T_\infty^{*3} T - 3T_\infty^{*4} \tag{8}$$

On using (7) & (8) in equation (3), we obtain the energy equation as follows

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^{*2}} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^{*2}} + \frac{\overline{Q}}{\rho C_p} \frac{\partial}{\partial y}(T - T_\infty) + R^*(C - C_\infty) \tag{9}$$

The following non-dimensional parameters are introduced in the equations (2), (4) & (9) to get governing equations in dimensionless form.

$$u = \frac{u^*}{U_0}, \quad y = \frac{y^* U_0}{v}, \quad t = \frac{t^* U_0^2}{v}, \quad n = \frac{n^* v}{U_0^2},$$

$$\Pr = \frac{\rho c_p v}{k}, \quad Sc = \frac{v}{D}, \quad M = \frac{\sigma B_0^2 v}{\rho U_0^2}, \quad K = \frac{k^* U_0^2}{v^2},$$

$$Gr = \frac{vg\beta(T_w^* - T_\infty^*)}{U_0^3}, \quad \theta = \frac{T - T_\infty}{T_w^* - T_\infty^*}, \quad k_r^2 = \frac{k_r^{*2} v}{U_0^2}, \tag{10}$$

$$\phi = \frac{C - C_\infty}{C_w^* - C_\infty^*}, \quad Nr = \frac{16\sigma^* T_\infty^3}{3k^* k}, \quad Q = \frac{\overline{Q} v}{\rho c_p U_0^2}$$

$$Gm = \frac{vg\beta^*(C_w^* - C_\infty^*)}{U_0^2}, \quad R = \frac{R^* v (C_w^* - C_\infty^*)}{\rho c_p (T_w^* - T_\infty^*) U_0^2}$$

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - \left(M + \frac{1}{k}\right) u \tag{11}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \left(\frac{1 + Nr}{\Pr}\right) \frac{\partial^2 \theta}{\partial y^2} + Q \frac{\partial \theta}{\partial y} + R\phi \tag{12}$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr^2 \phi \tag{13}$$

Where the parameters are thermal Grashof number *Gr*, modified Grashof number *Gm*, thermal radiation parameter *Nr*, heat source parameter *Q*, Schmidt number *Sc*, chemical reaction parameter Kr^2, Hartmann number *M*, porosity parameter *K*, Prandtl number *Pr*, Schmidt number *Sc* and Radiation absorption parameter *R*.

The corresponding initial and boundary conditions are

Case (I):

$$u = 0, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt} \ at \ y = 0$$
$$u \to 0, \theta \to 0, C \to 0 \ as \ y \to \infty \tag{14}$$

Case (II):

$$u = 1, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt} \ at \ y = 0$$
$$u \to 0, \theta \to 0, C \to 0 \ as \ y \to \infty \tag{15}$$

3. FORMULATION OF THE PROBLEM

The equations (11) to (13) are coupled non-linear partial differential equations whose solution in closed form are difficult to obtain, to solve these non-linear partial differential equations, The velocity, temperature and concentration fields are assumed in the following form:

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{nt} u_1(y) + 0(\varepsilon^2) \\ \theta &= \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + 0(\varepsilon^2) \\ \phi &= \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + 0(\varepsilon^2) \end{aligned} \tag{16}$$

We now substitute equation (16) in equations (11) to (13) and equating harmonic and non-harmonic terms, neglecting higher order terms in ε, we obtain:

$$u_0'' + u_0' - M_1 u_0 = -Gr\theta_0 - Gm\phi_0 \tag{17}$$

$$u_1'' + u_1' - (n + M_1) u_1 = -Au_0' - Gr\theta_1 - Gm\phi_1 \tag{18}$$

$$\theta_0'' + h(1+Q)\theta_0' = -Rh\phi_0 \tag{19}$$

$$\theta_1'' + h(1+Q)\theta_1' - nh\theta_1 = -hA\theta_0' - Rh\phi_1 \tag{20}$$

$$\phi_0'' + Sc\phi_0' - KrSc\phi_0 = 0 \tag{21}$$

$$\phi_1'' + Sc\phi_1' - Sc(Kr + n)\phi_1 = -ScA\phi_0' \tag{22}$$

Where $M_1 = M + \frac{1}{k}, \quad h = \frac{\Pr}{1 + Nr}$ and primes indicate differentiation with respect to y.

The boundary conditions are:

Case (I):

$$u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1,$$
$$\phi_0 = 1, \quad \phi_1 = 1 \quad at \quad y = 0$$

$$u_0 \to 0, \quad u_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0,$$
$$\phi_0 \to 0, \quad \phi_1 \to 0 \quad as \quad y \to \infty \tag{23}$$

Case (II):

$$u_0 = 1, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1,$$
$$\phi_0 = 1, \quad \phi_1 = 1 \quad at \quad y = 0$$

$$u_0 \to 0, \quad u_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0,$$
$$\phi_0 \to 0, \quad \phi_1 \to 0 \quad as \quad y \to \infty \tag{24}$$

The solutions of (17) to (22) under the transformed boundary conditions (23) and (24) yield

$$\phi_0 = e^{-m_1 y} \tag{25}$$

$$\phi_1 = B_1 e^{-m_1 y} + B_2 e^{-m_2 y} \tag{26}$$

$$\theta_0 = B_3 e^{-m_1 y} + (1 - B_3) e^{-m_3 y} \tag{27}$$

$$\theta_1 = B_4 e^{-m_1 y} + B_5 e^{-m_1 y} + B_6 e^{-m_3 y} + B_7 e^{-m_4 y} \tag{28}$$

$$u_0 = B_8 e^{-m_1 y} + B_9 e^{-m_3 y} + B_{10} e^{-m_5 y} \tag{29}$$

$$u_1 = B_{11} e^{-m_1 y} + B_{12} e^{-m_2 y} + B_{13} e^{-m_3 y}$$
$$+ B_{14} e^{-m_4 y} + B_{15} e^{-m_5 y} + B_{16} e^{-m_6 y} \tag{30}$$

Skin friction

The skin-friction coefficient at the plate is given by

$$\tau = \left[\tau_{xy} / \rho v_0^2\right]_{y=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{31}$$

Nusselt number

Rate of heat transfer in terms of Nusselt number at the plate is given by

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{32}$$

Sherwood number

Rate of mass transfer in terms of Sherwood number at the plate is given by

$$Sh = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{33}$$

4. RESULTS AND DISCUSSION

A study of velocity, temperature, concentration, skin-friction, heat transfer and mass transfer of the MHD mixed convection flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite surface with time dependent suction and temperature gradient heat source has been carried out, taking radiation and chemical effects into account. The numerical values of velocity, temperature, concentration, skin friction, heat and mass transfer are computed for cooled plate (Gr > 0).

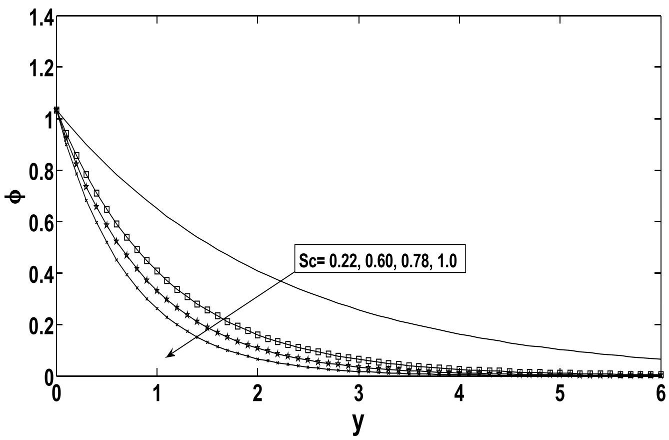


Figure 1: Concentration Profiles for various values of Sc

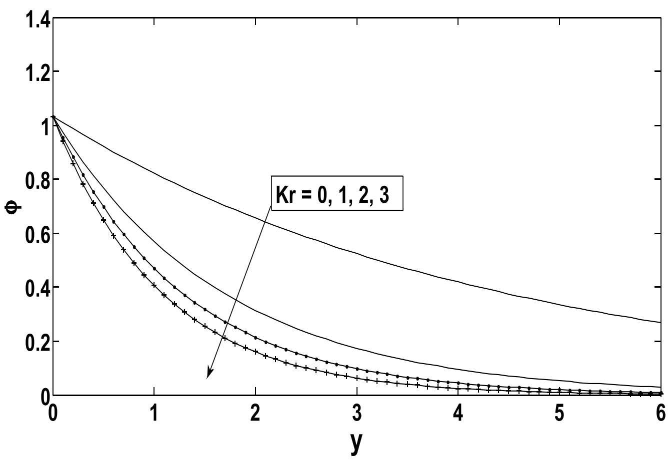


Figure 2: Concentration Profiles for various values of Kr

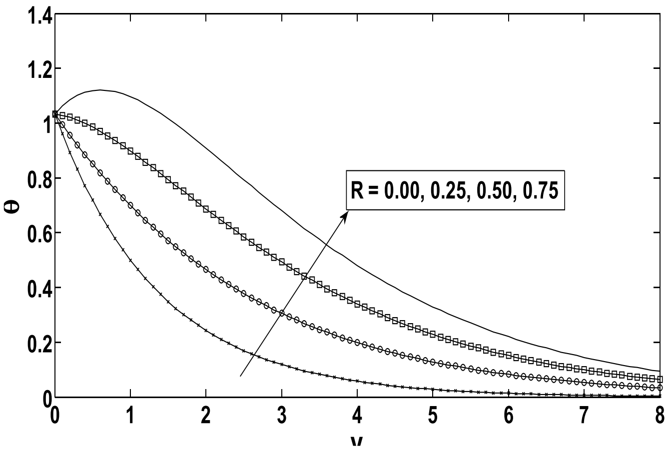


Figure 3: Temperature Profiles for various values of R

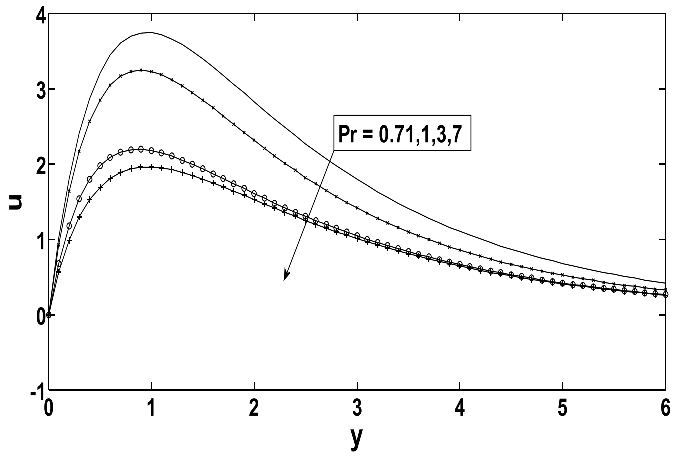


Figure 5: Velocity Profiles against y externally cooled plate with variations of Pr When Sc = 0.22, Gr = 10, Gm = 5, Q = 0.1, Nr = 0.1, Kr = 0.5, M = 0.5, k = 1, A = 0.3, t = 1 and R = 0.1

Figures 1&2 shows the effects of Schmidt number *Sc* and chemical reaction parameter *Kr* on concentration profiles. It is observed that an increase in Schmidt number *Sc* or chemical reaction parameter *Kr*, the concentration decreases. Also, it is noticed that the concentration boundary layer becomes thin as the Schmidt number or chemical reaction parameter increases. The effect of absorption radiation parameter R on the temperature field is shown in Figure 3. It is noticed that the temperature increases with an increase in absorption radiation parameter.

Case (I):

The fluid velocity variation in case of externally cooled plate is shown in figure 4 for various values of Schmidt number Sc. It is observed that for heavier diffusing foreign species i.e., increasing Schmidt number leads to a decrease in the velocity. Figure 5 depict the effect of Prandtl number Pr on the velocity field. It is noticed that an increase in Prandtl number leads to reduction in the velocity field.

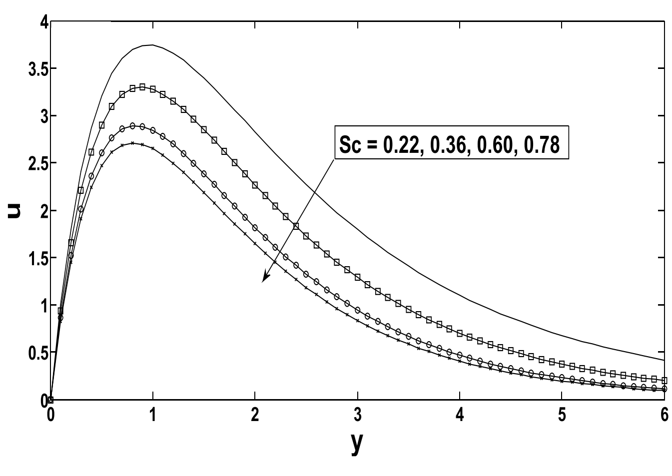


Figure 4: Velocity Profiles against y externally cooled plate with variations of Sc When Pr = 0.71, Gr = 10, Gm = 5, Q = 0.1, Nr = 0.1, Kr = 0.5, M = 0.5, k = 1, A = 0.3, t = 1 and R = 0.1.

Case (II):

Figure 6 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number. The velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then drops properly to approach the free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increases in the concentration buoyancy effects represented by Gm.

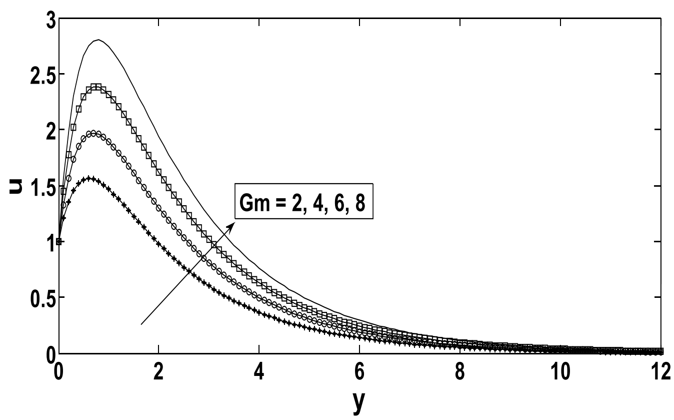


Figure 6: Velocity Profiles against y externally cooled plate with variations of Gm When Sc = 0.22, Gr = 10, Pr = 0.71, Q = 0.1, Nr = 0.1, Kr = 0.5, M = 0.5, k = 1, A = 0.3, t = 1 and R = 0.1

5. CONCLUSION

In this paper the effects of chemical reaction and radiation on unsteady MHD free convection fluid flow embedded in a porous medium with time dependent suction and temperature gradient heat source has been analyzed in two different cases of boundary conditions namely (case1) Flow past a stationary plate with variable temperature and concentration and (case2) Flow past a moving plate with variable temperature and concentration. From our results, the following conclusions are drawn.

1. The increase of Schmidt number and chemical reaction parameter both decelerates the concentration of the fluid.
2. Temperature increases as radiation absorption parameter R increases.
3. In the case of flow past a stationary plate, velocity profiles decrease with increase in Schmidt number or Prandtl number.
4. In the case of flow past a moving plate, increase of mass Grashof number Gm enhances the velocity of the fluid flow.

ACKNOWLEDGEMENT

The authors thank their respective departments of Mathematics at the RVR & JC College of Engineering, Guntur and S.V. University, Tirupati, Andhra Pradesh, India for the research-stimulating environment.

REFERENCES

[1] Chambre, P.L and Young, J.D, On the diffusion of chemical reactive species in a laminar boundary flow, the physics of fluids, 1, (1990), 48-54.

[2] Chamkha, A.J, Thermal radiation and buoyancy effects on hydro-magnetic flow over an accelerating permeable surface with heat source/sink, Int.J. Heat and mass transfer, 38, (2000), 1699-1712.

[3] Chen, Heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration, Acta Mechanica, 172(3), (2004), 219-235.

[4] Cookey. C.I, Ogulu. A & Omubo-pepple. V. B, Influence of viscous dissipation and radiation on unsteady MHD free convective flow past an infinite heated vertical plate in a porous medium with time-dependent suction, International Journal of Heat and Mass Transfer, 46(13), (2003), 2305-2311.

[5] Cramer, K.R and Pai, S.L, Magneto Hydrodynamics for Engineers and Physicists, Mc Graw-Hill, New York. (1975).

[6] Das, U.N, Dekha, R.K and Soundalgekar, V.M, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, The Bulletin, GUMA, 5, (1999), 13-20.

[7] Hady, F.M and Hossaine, J.A, Heat and mass transfer for power-law fluids, Indian J. Pure Appl. Maths, 17(1), (1986), 108-120.

[8] Ogulu, A. and Mbeledogu. I. U, Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer, International Journal Heat Mass Transfer, 50.(2007), 1902-1908.

[9] Prakash, J. and Ogulu. A, Unsteady two-dimensional flow of a radiating and chemically reacting MHD fluid with time-dependent suction, Indian Journal of Pure Applied Physics, 44, (2006), 805-810.

[10] Raptis, A. Radiation and free convection flow through porous medium. International Communications in Heat and Mass Transfer, 25(2), (1998), 289-295.

[11] Sunitha. S. Bhasker Reddy, N.and Rama Chandra Prasad. V, Radiation and mass transfer effects on MHD free convective flow past an impulsively started isothermal vertical plate with dissipation, Thermal Science, 13(2), (2009), 71-81.

[12] Shanker, B. Prabhakar Reddy, B. and Ananda Rao J, Radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption, Indian Journal of Pure Applied Physics, 4, (2010), 157-167.

[13] Sudharsan Reddy, Y. Varma S.V.K, Balamurugan K.S and Rama Prasad J.L, Chemical reaction and radiation absorption effects on hydro magnetic free convection flow past a vertical plate with constant mass flux, Far East J. Math. Sci. (FJMS), 98(2), (2015), 133-149.

[14] Balamurugan K.S, Rama Prasad J.L and Vijaya Kumar Varma S, Unsteady MHD free convective flow past a moving vertical plate with time dependent suction and chemical reaction in a slip flow regime, Procedia Engineering, 127, (2015), 516-523.

[15] Vijaya Kumar, A.G, Rushi Kumar, B, Balamurugan K.S and Varma, S.V.K, The effects of induced magnetic field and radiation on MHD convective chemically reacting fluid over a porous vertical plate, Far East J. Math. Sci. (FJMS), 80(1), (2013), 135-154.

[16] Seshaiah, B, Varma, S.V.K and Raju, M.C, The effects of chemical reaction and radiation on unsteady MHD free convective fluid flow embedded in a porous medium with time dependent suction with temperature gradient heat source, Int. J. Sci. Knowledge Computing Information Tech., 3(2), (2013), 13-24.

◆ ◆ ◆