ADMP: A Maple Programming Language for Symbolic Computation to Nonlinear Quadratic Riccati Differential Equation

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Abstract: **In this paper, we apply an analytical method called the Two Step – Adomian Decomposition Method joint with Padè – approximation (TSADM – Padè) for solving various nonlinear Riccati differential equations. The process for accurate analytic approximation of nonlinear differential equations along with some boundary conditions, uses the new definition of Adomian polynomials. For this purpose a Maple package ADMP is used.**

Key Words: **Two-Step Adomian-Decomposition Method (TSADM), Padè approximants, nonlinear Riccati equation, approximate solution, ADMP.**

1. INTRODUCTION

The method known as the Adomian Decomposition Method (ADM) was firstly introduced by G. Adomian in 1980 [1-3], for solving various linear and nonlinear functional equations arising in different fields of science and engineering. In this technique a nonlinear functional equation is decomposed in a series of functions by means of Adomian polynomials. The ADM is mainly applied to get approximate solutions for real world problems in science and engineering without using unphysical restrictive assumptions [4-13].

In the present paper we will use the following nonlinear Riccati equation:

$$\frac{du}{dt} = P(t) + Q(t)u(t) + R(t)u^2(t) \tag{1}$$

with

$$u(0) = \alpha \tag{2}$$

Where P, Q and R are continuous functions of t and α is a constant. Earlier so many methods are applied to solve this kind of Riccati equation.. In the present paper we implement an efficient numerical technique in which the solution of the problem with various boundary conditions transformed from the developed computer algorithms to the approximate solution, that converges to the exact solution. This technique was developed by Y.Lin [14].

2. THE PACKAGE ADMP

For various nonlinear differential equations with initial / boundary conditions, the MAPLE package ADMP [14] automatically derives the analytic approximate solutions in graphical and numerical form.

The main interface is *Asolve* (*Eqs, IBCs, [funcs], ord, extra_args*), where the parameter *Eqs* is a list of differential equations, *IBCs* represents a list of initial and boundary conditions, *funcs* is a list of unknown functions of the input system, *ord* is a list of fractional order of the input system, *extra_args* is optional, depending on the sophistication of our physical model. The *extra_args* includes *index=k*, *class=d*, *Padè=[m, n]*, and *output =plot*, or a name of the file, where *k* is the required highest order of the truncated series solutions, here *d* represents the class number of the Adomian polynomials, and *m, n* are respectively the desired degree of the numerator and denominator of the Padé approximation. The default *index* is 5, the default *class* is 4, the default *Padè* is [3, 3], and the default *output* is *plot*.

3. THE APPLICATION OF ADMP

In this section, an example is given to illustrate the algorithm and demonstrate the effectiveness of the package to note the completion time of the process ADMP.

Example 1 Consider the following nonlinear riccati differential equation

$$\frac{du}{dt} = 1 + 2u(t) - u^2(t) \tag{3}$$

with the initial conditions $u(0) = 0$

The exact solution of equation (3) together with initial condition is given by

$$u(t) = 1 + \sqrt{2}\, tanh\left[\sqrt{2t} + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right] \tag{4}$$

Using *class =4* and *index =25*, the package automatically delivers the solutions of 25th order approximations and 22nd order approximations by the TSADM in Fig.1, completed the output in 0.302 s.

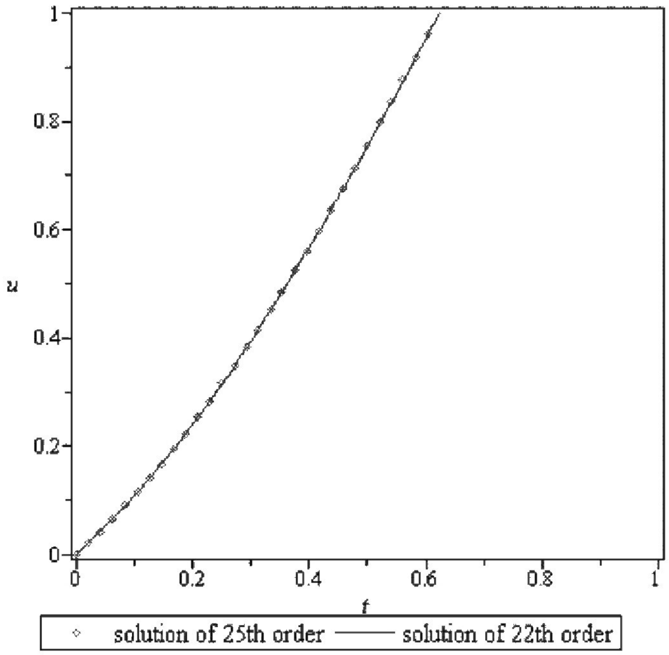

Fig 1. Approximate solution graph of 25th order and 22nd order by two step Adomian decomposition method (TSADM) for $0 < t \le 1$

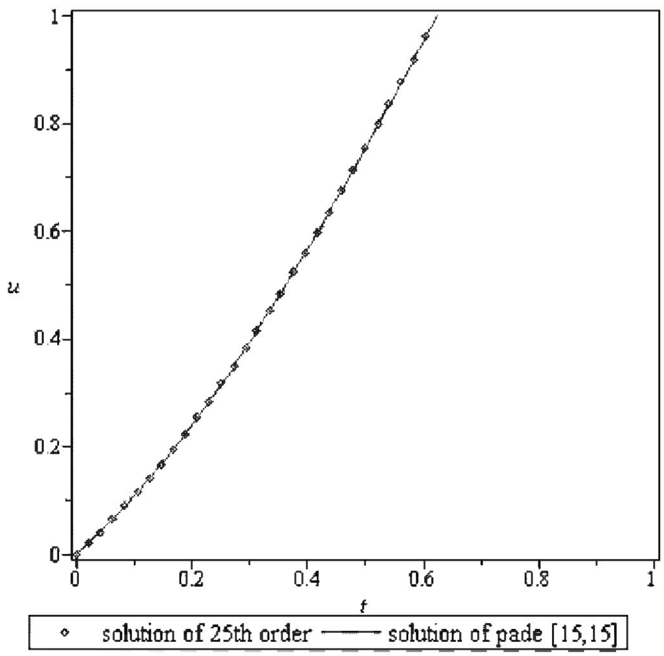

Fig.2 Comparison of 25th order TSADM with Combined TSADM- Padè approximants [15, 15] for $0 < t \le 1$

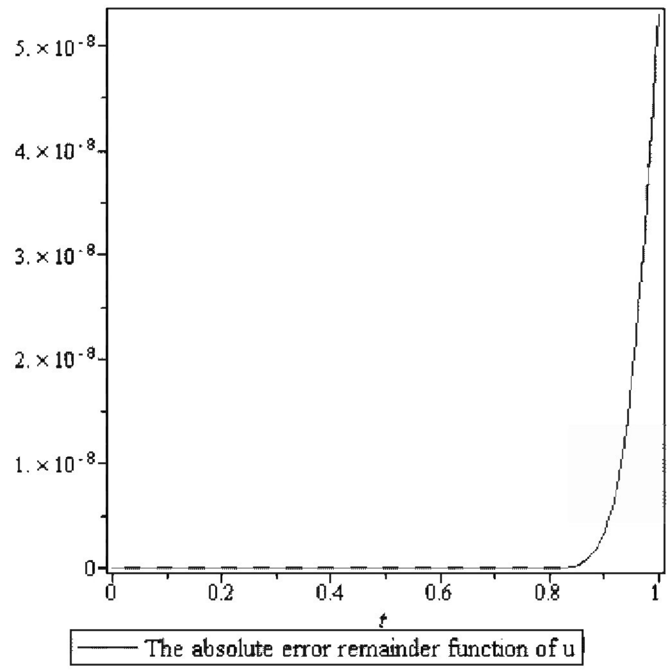

Fig.3 The absolute error of $|u_{exact} - \varphi_{40}(t)|$ for $0 < t \le 1$

The expression of the Padé [15/15]:

Padé [15/15]=$[(0.00015\, t^{15} - 0.14219610\, t^{14} + 0.3019642\, t^{13} - 68.4309455\, t^{12} + 94.1094278\, t^{11} - 8375.93306\, t^{10} + 9655.62637\, t^{9} - 3.58070\; 10^{5} t^{8} + 3.801903.58070\; 10^{5} t^{7} - 5.22179\; 10^{6} t^{6} + 5.34379\; 10^{6} t^{5} - 1.80142\; 10^{7} t^{4} + 1.82175\; 10^{7} t^{3} + 4411.8039\, t^{2} + t)/(4410.80396\, t + 1.82130\; 10^{7} t^{2} - 3.62287\; 10^{7} t^{3} + 3.55030\; 10^{7} t^{4} - 2.25754\; 10^{7} t^{5} + 7.54518\; 10^{6} t^{6} - 7.30026\; 10^{-9} t^{7} + 5.98915\; 10^{-10} t^{8} - 2.07604\; 10^{-11} t^{9} + 1.1371810\; 10^{-12} t^{10} - 1325.86548\, t^{11} + 77.2883\, t^{12} - 5.77643\, t^{13} + 0.15561\, t^{14} - 0.00379\, t^{15})]$

By using the two step Adomian decomposition method (TSADM) the approximations are as follows

$\varphi_0(t) = 0,$

$\varphi_1(t) = t,$

$\varphi_2(t) = t^2,$

$\varphi_3(t) = \frac{t^3}{3}$,

And so on

Hence the exact solution is given by

$$u = \sum_{p=0}^{\infty} \varphi_p(t) = 1 + \sqrt{2}\, tanh\left[\sqrt{2t} + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right] \quad (5)$$

The ADMP delivers outputs Fig.2, in 25th order TSADM with Combined TSADM- Padè approximants [15, 15] for $0 < t \leq 1$. Also Fig.3 represent the absolute error between the exact solution and 40th-order approximate solution gives better approximation by TSADM.

Appendix A. The use of the Package ADMP

To load the ADMP package, one can proceed as follows:

> restart ; initializing Maple.

> read "*Adomian.mpl*"; reading the program file into memory.

> with(ADMP) ; loading the package ADMP.

Consider the example 1, one can run the main procedure as follows:

> eq:=$diff(u(t),t) = 16 * t\$2 - 5 * t + 8 * t * u(t) + u(t)^2$;

>ICs:=$[u(0) = 1]$;

>*Asolve ([eq], ICs, index=20, u=0..1, t=0..1, output=plot, Padè = [10, 10]);*

The package automatically delivers the following results as shown in the example 1

Appendix B. The procedures contained in the software package ADMP

The software package ADMP is comprised of 20 different procedures, among which are *Asolve, AdomianProcess, CaputoD, DefineVars, GetIBCs, DefineFdepvars, DefineIntvars, DefineOperator, SeparateOperator, Aplot, SpecOperator, DefineInverseOperator, SystemPrint, 'ADM/process', CallntCon, DetermineAdomian, InputValue, cmpplot, ErrorAnalysis and ErrorPlot.*

- *Asolve*: This is the main procedure; its main task is to call other procedures.
- *AdomianProcess*: The main task is to call the subroutine *'ADM/process'* and receives the input parameters.
- *DefineVars,DefineFdepvars,DefineIntvars*: The main task of this procedure is to initialize relevant variables.
- *GetlBCs*: The main task of this procedure is to redefine initial or boundary conditions.
- *DefineOperator ,SeparateOperator, SpecOperator, Define InverseOperator:* The main task of these procedures is to define the linear as well as inverse operator, nonlinear operators , remainder operators and source terms.
- *'ADM/process':* The main task of this procedure is to call *CallntCon* and *DetermineAdomian* to compute the Adomian polynomials, and construct analytic approximate solutions of the input system.
- *CallntConI*; The main task of this procedure is to compute the initial term u_0.
- *DetermineAdomian:* The main task of this procedure is to show the partial sums of the Adomian polynomials based on the different classes, which is controlled by the parameter *class*.
- *SystemPrint*: The main task of this procedure is to output the equations and initial/ boundary conditions.
- *InputValue*: The main task of this procedure is to receive some unknown parameters.
- *Aplot, cmpplot*: The main task of this procedure is to output the corresponding graphs.
- *ErrorAnalysis, ErrorPlot*: The main task of this procedure is to output the corresponding graphs.
- *CaputoD:* The main task of this procedure is to define the Riemann-Liouville integral and Caputo derivative.

4. CONCLUSION

In this paper, a Maple software package ADMP has been successfully applied to find an approximate solution of nonlinear Riccati differential equation. The effectiveness and efficiency of the package has been depicted through example. Meanwhile, it also displays graph of comparison of different order approximation, their respective Padè approximants and graphs for error analysis.

REFERENCES

[1] G. Adomian, Solving Frontier Problems of Physics: The Decomposition method, Kluwer Academic Publishers, 1994.

[2] G.Adomian, Nonlinear Stochastic operator equations. Academic press, San Diego 1986.

[3] G.Adomian, G.E.Adomian, A global method for solution of complex systems, Mathematical Modelling, 5, 521-568, 1984.

[4] S.M.El-Sayed, D.Kaya, Exact and numerical travelling wave solution of Whitham-Broer-Kaup equations, Applied Mathematics and Computation 167 (2005) 1339-1349.

[5] L.Bougoffa, S.Bougoffa, Adomian method for solving some coupled systems of equations, Applied Mathematics and Computation, 177 (2006) 553-560.

[6] M.Danish, S.Kumar, S.Kumar, Approximate explicit analytical expressions of friction factor for flow of Bingham fluids in smooth pipes using Adomian-Decomposition method, Communication in Non Linear Sciences and Numerical Simulations 16 (2011) 239-251.

[7] S.Abbasbandy, Improving Newton-Raphson method for nonlinear equations by Modified Adomian Decomposition method, Applied Mathematics and Computation 145 (2003) 887-893.

[8] A.M.Wazwaz, A comparison between Adomian-decomposition method and Taylor's series method in series solution, Applied Mathematics and Computation 197 (1998) 37-44.

[9] A.Saravanan, N.Magesh, A comparison between reduced Differential transform method and Adomian decomposition method for the Newell-Whitehead-Segel equation, Journal of Egyptian Mathematical Society, 21 (3) (2013) 259-265.

[10] M.A.Noor, K.I.Noor, M.Waseem, A new decomposition method for solving a system of linear equation, Journal of Association Arab Universities for Basic and Applied Sciences,10, (2013) , 421-429.

[11] V.D.Gejji, H.Jafari, Adomian Decomposition Method: a tool for solving a system of fractional differential equations, Journal of Mathematical Analysis and Applications, 301 (2005) 508-518.

[12] R.Singh, G.Nelakanti, J.Kumar, Approximate solution of Urysohn Integral equation using the Adomian decomposition method, The World Scientific Journal, (2014) Article ID 150483, 6 pages.

[13] B.Jang, Solving two-point boundary value problems by the extended Adomian decomposition method, Journal of Computational and Applied Mathematics, 219 (1) (2008) 253-262.

[14] Y.Lin, Y.Liu, Z.Li, Symbolic computation of analytic approximate solution of nonlinear fractional differential equations, Computer Physics Communications, 184 (2013), 130-141.

[15] K. Abbaoui, Y. Cherrault, Convergence of Adomian's method applied to differential equations, Computers and mathematics with Applications, 28 (5) (1994) 103-109.

[16] M.Benabidallah, Y.Cherrault, Application of Adomian method for solving a class of boundary problems, Kybernetes, 33 (1) (2004) 118-132.

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