A Weighted Least Square Technique: For Assessment of State Estimation of Power System

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Abstract: **As Power system grows larger and more complex real time monitoring and control becomes very significant to achieve reliable operation of power system. Energy management system (EMS) functions are responsible for the task of monitoring and control. State estimation (SE) forms the backbone of the EMS by providing real time data base of the state of the system for using in the other EMS function. Hence an accurate & efficient state estimation is a pre-requisite for an efficient and reliable operation of power system. This paper present study of Weighted Least Square (WLS) method for 6-bus sample power system and IEEE 30-bus power system and our data show its effectiveness.**

Keywords— **Energy management system(EMS), IEEE 30-bus standard load flow test, State Estimation (SE), Weighted Least Square (WLS) method.**

1. INTRODUCTION

A power network is a intricate system connecting electric power generators to consumers through power transmission and distribution networks over a large area. Monitoring and control of a complex interconnected power system requires the accurate estimation of its states. Different meters are placed at the various substations and the measurements are transmitted to the central control centre. However, there may be errors associated with the measurements and in some cases some measurements may not be available [1].So that state estimation becomes one of the most reliable modules in EMS(Energy Management System) as well as indispensably tool for the system operator.

There are different types of methods which are used in state estimation of power system i.e. Weighted Least Square State estimation, Normal equation method, Orthogonal transformation, Hybrid method, Normal equation method with constraints, Hachtel's augmented matrix method, Observabilty Analysis, Bad Data Detection. Comparison of these methods are done in the terms of Numerical stability, computational efficiency, implementation complexity [2, 3].

State estimation is a technique developed to provide an estimate of an unknown system state variable. In state estimation of power system, state estimator is used. State estimator is a data processing algorithm for converting redundant meter readings and other available information into an estimate of the state of an electric power system. It plays an essential part in every energy management system and also is a basic tool in ensuring the secure operation of a power system. The obtained calculated method of state estimation of power system is then used to solve various technological problems and effectively control the electric power system. In doing so the necessity arises based on a statistical criterion that estimates the true value of the state variables to minimize or maximize the selected criterion. The most widely used criterion is that of minimizing the sum of the squares of the differences between the estimated and "true" (i.e., measured) values of a function [4-5].

This paper describes State Estimation methods i.e. Weighted Least Square method for estimation of power system, doing so MATLAB is used. The proposed method has been tested on 6 bus sample power system and IEEE 30-bus power system.

The organization of paper is follows: Section II explain the role of state estimation in the power system including the function of state estimator. Section III describe about the mathematical formulation of WLS method. Section IV represent the result obtained for 6-bus sample power system and IEEE 30-bus power system.

2. STATE ESTIMATION IN POWER SYSTEM

State estimation (SE) provides a database to the operators to take the necessary and efficient action to control the contingencies. Basic role of SE is generation of coherent and reliable real time data base from the information provided by remote signaling and measurements. SE plays a key role in real time power system operation. Best estimate of the current actual power system states (voltage, angle, Circuit breaker (CB) status, taps position etc.) are determined by the SE based on supervisory control and data acquisition (SCADA) measurements, power system model and other data. SE results are then used in network contingency analysis, security enhancement, optimal power flow, dynamic security analysis (including voltage and transient stability) and other applications [6-7].

System monitoring is necessary to ensure the reliable operation of power grids, and state estimation is used in system monitoring to best estimate the power grid state through analysis of meter measurements and power system models [8].

Power system state estimation is a systematic procedure or mathematical procedure to process the set of real time measurement for to obtain the best estimate of the current states of power system. It utilizes redundant measurement from the

SCADA system to compute the online states of bus in an estimator. State estimation is a technique developed to provide an estimate of an unknown system state variable and to quantitatively analyze the estimated state variable before it is used for real time power -now calculations or on-line system security assessment. It plays an essential part in every energy management system and also is a basic tool in ensuring the secure operation of a power system [9].

3. WEIGHTED LEAST SQUARE METHOD

This section describes the conventional WLS State Estimation equation in order to introduce basic concepts and notations. The non-linear equations relating the measurements and the state vector [10] are as follows,

$$z = h(x_t) + w \tag{1}$$

Where z is the (m x 1) measurement vector, h (x) is the (m x 1) vector of non-linear functions, x is the (n x 1) true state vector, w is (m x 1) measurement error vector, number of measurements represented by the m and number of state variable is represented by the n

The estimate of the unknown state vector xt is designated by $\hat{x}$ and is finding out by the minimizing the least squares function.

$$J(x) = [z - h(x)]^T W[z - h(x)] \tag{2}$$

Where,

W = diagonal matrix whose elements are the inverses of the measurement variances, i.e.

$$W = [cov(w)]^{-1} \tag{3}$$

The condition for optimality is that the gradient of J vanishes at the optimal solution $\hat{x}$ i.e.

$$H^T(\hat{x})W[z - h(\hat{x})] \tag{4}$$

Where, the Jacobian matrix H(x) is,

$$H(x) = \frac{\partial h}{\partial x}(x) \tag{5}$$

The estimate $\hat{x}$ is obtained by solving the non-linear system $\partial j/\partial x = 0$ through the iteration process:

$$G(x^k)\Delta x^k = H^T(x^k)W\left(z - h(x^k)\right) \tag{6}$$

$$x^{k+1} = x^k + \Delta x^k \tag{7}$$

For k = 0, 1, 2.... until appropriate convergence is attained. Here the gain matrix G is,

$$G = H^T(x^k)WH(x^k) \tag{8}$$

The estimation residuals are defined as:

$$r = z - h(\hat{x}) \tag{9}$$

The covariance matrix of the residual vector r is given by,

$$cov(r) = R = W^{-1} - H(\hat{x})G^{-1}(\hat{x})H^T(\hat{x}) \tag{10}$$

The sensitivity matrix is given by,

$$\frac{\partial \hat{x}}{\partial Z} = G^{-1}(\hat{x})H^T(\hat{x})W \tag{11}$$

And the sensitivity matrix is given by [4]:

$$\frac{\partial r}{\partial z} = 1 - H(\hat{x})G^{-1}(\hat{x})H^T(\hat{x})W = RW \tag{12}$$

The flowchart of WLS method is shown in figure-1,

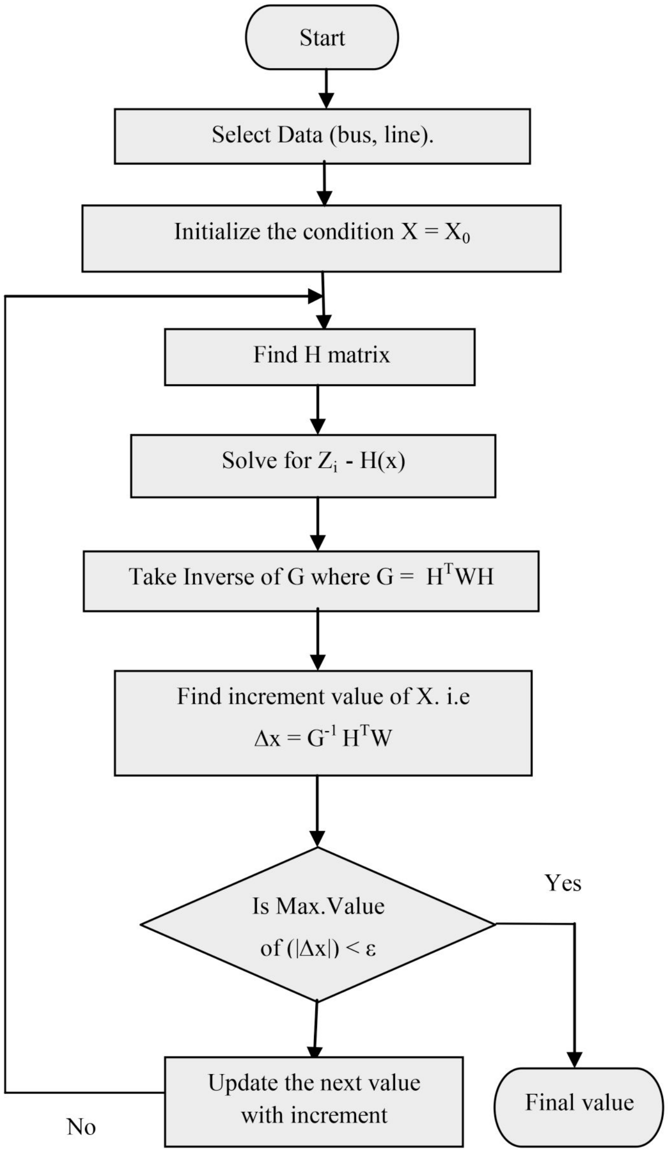

Fig. 1.Flowchart of WLS method

Absence of state estimation solution causes the occurrence of cascading failures or blackouts in local and/or regional areas for considerable time periods. The robustness and reliability of state estimation is a critical issue for power utilities. The Weighted Least Square (WLS) method is the commonly used state estimation approach which is used in power industries [11-12]. In this paper weighted least square algorithm along with flow chart is presented and applied for 6-bus sample system and for IEEE 30-bus System. Results of these test systems have been discussed.

4. SIMULATION RESULTS

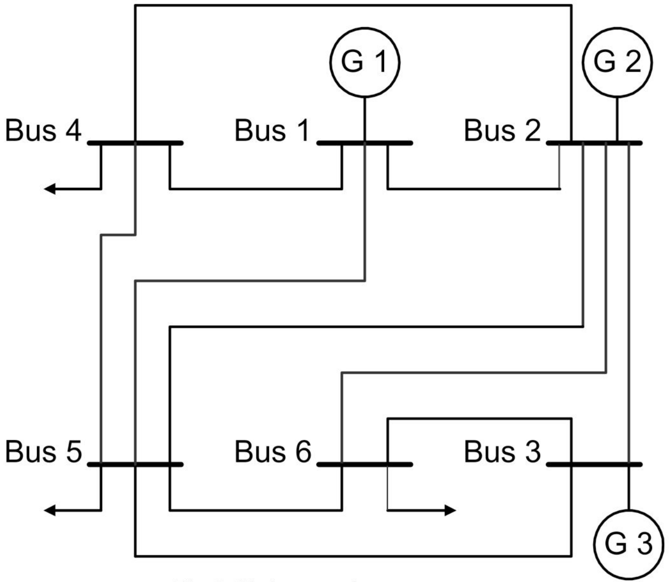

Fig. 2. Six-bus sample power system.

Simulation has been done in MATLAB. Simulated test data for 6-bus sample system [9] is shown in figure-2, has 19 number of measurements (Nm) (shown in table-I) and there are 11 state variables (NS = 2N-1), where N represents number of buses. So redundancy or degree of freedom is (Nm-NS) i.e. 8, (N-1) angles i.e. 5 and N voltage magnitudes i.e. 6 are determined in this test. Results of test for 6 bus sample system are represented in table-II. In this table column 2 and 3 represent the result obtained from Newton - Raphson load flow while column 4, 5, 6 and 7 give the result from WLS method. Figure-3 represents the comparison of true and estimated value (when noise in all measurements) of bus voltages for 6-Bus sample power system.

Table I. Measurements With And Without Noise For 6-bus Sample System

Sr. No.		Measurement without Noise	6% Noise in all measurements
1	Voltage Magnitude	1.05	1.113
2	Real Power Injection	1.078	1.1427
3		0.5	0.53
4		0.6	0.564
5		-0.7	-0.742
6	Reactive Power Injection	0.0159	0.1685
7		0.743	0.7876
8		0.896	0.9498
9		-0.7	-0.742
10	Real Power Flow	0.2869	0.3041
11		0.0293	0.0311
12		0.1911	0.2026
13		0.0408	0.0383
14		0.0161	0.0171
15	Reactive Power Flow	-0.1321	-0.14
16		-0.0896	-0.095
17		0.2603	0.2447
18		-0.0102	-0.0108
19		-0.0675	-0.0716

Table II. Estimated States With And Without Noise For 6-bus Sample System

Bus No.	Measured Voltage (p.u)	Measured Angle (Degree)	Measurements without noise		6% Noise in all measurements	
			Estimated V (p.u)	Estimated Angle (Degree)	Estimated V (p.u)	Estimated Angle (Degree)
1	1.05	0	1.0387	0	1.0974	0
2	1.05	-3.6712	1.0376	-3.7014	1.0956	-3.5376
3	1.07	-4.2733	1.0526	-4.2402	1.1101	-4.213
4	0.9894	-4.1958	0.984	-4.4504	1.0409	-4.1618
5	0.9854	-5.2764	0.9641	-5.1852	1.0241	-4.9302
6	1.0044	-5.9475	0.9858	-5.963	1.0437	-5.7348

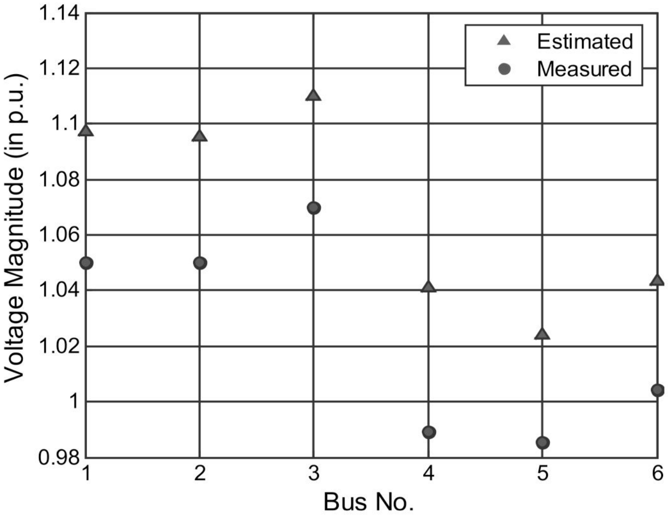

Fig.3 Comparison of true and estimated value (when noise in all measurments) of bus voltages for 6-Bus sample power system

Similarly simulated test data for IEEE 30-bus system [8] has 93 number of measurements (Nm) (shown in table-III) and there are 59 state variables (NS = 2N-1), where N represents number of buses. So redundancy or degree of freedom is (Nm-NS) i.e. 34, (N-1) angles i.e. 29 and N voltage magnitudes i.e. 30 are determined in this test. Results of test for IEEE 30-bus system are represented in table-IV. In this table column 2 and 3 represent the result obtained from Newton - Raphson load flow while column 4, 5, 6 and 7 give the result from WLS method. Figure-4 represents the comparison of true and estimated value (when noise in all measurements) of bus voltages for 30-Bus sample power system.

Table III. Measurements With And Without Noise For IEEE 30-bus System

Sr. No.		**Measurement without Noise**	**Noise in all measurements**
1	Voltage Magnitude	1.06	1.1236
2	Real Power Injection	-0.076	-0.0806
3		-0.942	-0.9985
4		0	0
5		-0.3	-0.282
6		-0.058	-0.0615
7		0	0
8		0	0
9		-0.062	-0.0657
10		-0.082	-0.0869
11		-0.035	-0.0371
12		-0.032	-0.0339
13		-0.022	-0.0233
14		-0.175	-0.1855
15		-0.087	-0.0922
16		0	0
17		-0.035	-0.0371
18		0	0
19		-0.024	-0.0254
20	Reactive Power Injection	-0.016	-0.017
21		0.166	0.176
22		0	0
23		0.061	0.0647
24		-0.02	-0.0188
25		0.16	0.1696
26		0.104	0.1102
27		-0.016	-0.017
28		-0.025	-0.0265
29		-0.018	-0.0191
30		-0.009	-0.0095
31		-0.007	-0.0074
32		-0.112	-0.1187
33		-0.067	-0.071
34		0	0
35		-0.023	-0.0244
36		0	0
37		-0.009	-0.0095
38	Real Power Flow	1.7331	1.8371
39		0.8765	0.9291
40		0.4365	0.4627
41		0.8236	0.873
42		0.6038	0.64
43		0.7213	0.7646
44		0.3813	0.4042
45		0.2956	0.3133
46		0.2772	0.2938
47		0	0
48		0.2772	0.2938
49		0.4419	0.4684
50		0	0
51		0.1789	0.1896
52		0.0158	0.0167
53		0.0369	0.0391
54		0.0278	0.0295
55		-0.0673	-0.0713
56		0.0903	0.0957
57		0.1579	0.1674
58		0.0762	0.0808
59		-0.0183	-0.0194
60		0.0574	0.0608
61		-0.0121	-0.0128
62		-0.0476	-0.0447
63		0.1807	0.1915
64		0.037	0.0392
65		0.1867	0.1979
66	Reactive Power Flow	-0.247	-0.2618
67		0.0428	0.0454
68		0.0475	0.0503
69		0.0278	0.0295
70		0.0137	0.0145
71		-0.1591	-0.1686
72		-0.0278	-0.0295
73		-0.072	-0.0763
74		-0.0809	-0.0858
75		-0.156	-0.1654
76		0.0588	0.0623
77		0.1441	0.1527
78		-0.1032	-0.1094
79		0.0679	0.072
80		0.0065	0.0069
81		0.0144	0.0153
82		0.0062	0.0066
83		-0.0279	-0.0296
84		0.0371	0.0393

85	Reactive Power Flow	0.1001	0.1061
86		0.046	0.0488
87		-0.0143	-0.0152
88		0.0306	0.0324
89		0.0201	0.0213
90		-0.0037	-0.0039
91		0.0504	0.0534
92		0.0061	0.0065
93		0.0011	0.0012

Table IV. Estimated States With And Without Noise For IEEE 30-bus System

Bus No.	Measured Voltage (p.u)	Measured Angle (Degree)	Measurements without noise		6% Noise in all measurements	
			Estimated V (p.u)	Estimated Angle (Degree)	Estimated V (p.u)	Estimated Angle (Degree)
1	1.06	0	1.0597	0	1.123	0
2	1.045	-5.378	1.0442	-5.4195	1.1071	-5.1099
3	1.021	-7.529	1.0262	-8.068	1.0888	-7.6055
4	1.012	-9.279	1.0081	-9.5013	1.0703	-8.9432
5	1.01	-14.149	1.008	-14.2974	1.0706	-13.4639
6	1.011	-11.055	1.0048	-11.2285	1.067	-10.565
7	1.003	-12.852	0.9995	-13.0811	1.0615	-12.3107
8	1.01	-11.797	1.0047	-11.9888	1.067	-11.2652
9	1.051	-14.098	1.0179	-14.3823	1.0801	-13.5423
10	1.045	-15.688	1.0078	-16.1052	1.0699	-15.1625
11	1.082	-14.098	1.0498	-14.3823	1.1119	-13.5423
12	1.057	-14.933	0.9961	-15.4625	1.0581	-14.5446
13	1.071	-14.933	1.0106	-15.4625	1.0725	-14.5446
14	1.043	-15.825	0.9828	-16.4176	1.0448	-15.4412
15	1.038	-15.916	0.981	-16.5662	1.043	-15.5814
16	1.045	-15.515	1.0044	-16.1436	1.0661	-15.1879
17	1.04	-15.85	1.0101	-16.4799	1.0717	-15.5059
18	1.028	-16.53	0.9828	-17.1015	1.0449	-16.0928
19	1.026	-16.704	0.9843	-17.2487	1.0464	-16.2338
20	1.03	-16.507	0.9893	-17.0283	1.0514	-16.0276
21	1.033	-16.131	0.9955	-16.5862	1.0576	-15.614
22	1.034	-16.116	0.9961	-16.5716	1.0582	-15.6002
23	1.027	-16.307	0.9744	-17.1135	1.0364	-16.0879
24	1.022	-16.483	0.9774	-16.9336	1.0395	-15.9315
25	1.018	-16.055	0.9773	-16.4751	1.0392	-15.5009
26	1	-16.474	0.9613	-16.9466	1.0237	-15.9035
27	1.024	-15.53	0.9837	-15.9026	1.0451	-15.0048
28	1.007	-11.677	1	-11.8411	1.062	-11.1445
29	1.004	-16.759	0.963	-17.2342	1.0245	-16.2523
30	0.992	-17.642	0.951	-18.192	1.0126	-17.1477

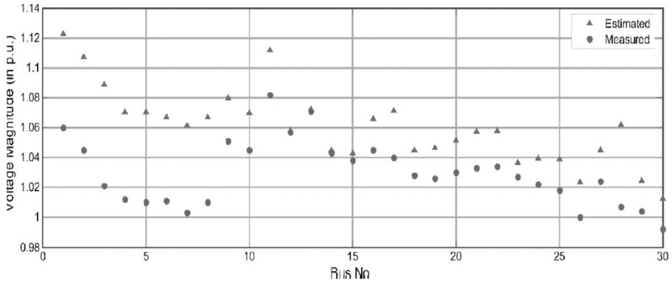

Fig.4 Comparison of true and estimated value (when noise in all measurments) of bus voltages for IEEE 30-Bus sample power system

5. CONCLUSIONS

The objective of the weighted least-squares (WLS) criterion is to minimize the sum of the squares of the weighted deviations of the estimated measurements , from the actual measurements z. The Weighted Least Square technique has been tested on 6-bus sample power system and IEEE 30-bus power system. The result was found that in both case the estimated values were very close to the actual values.

WLS state estimation technique is able to perform wall and provide a reliable estimate of state variable even with 6% noise is added in all measurement. For each system graphical representation is used for comparison of result in terms of bus voltage magnitude. The best estimates of system are obtained by the Weighted Least Square (WLS) state estimation method is demonstrated by the result i.e. accuracy in state estimation of system is obtained by the WLS state estimation method.

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