Design of a Channel Equalizer based on QR-RLS Algorithm in QAM Signals

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Received 23 January 2015, received in revised from 13 April 2015, accepted 13 April 2015

Abstract: **In Wireless Communication adaptive equalizer is play an important role for signal transmission. In this paper we present an advanced channel equalization system which based on QRD based adaptive (QR-RLS) filter algorithm and can use in Quadrature amplitude modulation (QAM) signals. The algorithm has properties that it can converge with high speed of convergence rate and gives minimum mean square error (MSE) compared to LMS, NLMS and RLS algorithms. An audio signal with noise is used to analyze the equalizer. The MATLAB results show that the proposed equalizer is how much effective with improved convergence rate and reduced MSE error especially with the QAM constellations.**

Key Words: **Adaptive Equalizer; Mean Square Error (MSE), Quadrature Amplitude Modulation (QAM), Least Mean Square (LMS), Normalized LMS (NLMS), Recursive least Square (RLS), Quadrative Recursive RLS (QR-RLS).**

1. INTRODUCTION

In wireless data communication Multi-path effect exists in transmission of signals. Different type of delay through different paths of signal is received in the same time, which produces a problem called Inter-Symbol Interference (ISI). This ISI also changes with time due to moving communications carrier and surrounding obstacles (like Huge buildings, moving vehicles etc.), which result in the dissemination of changes in environmental with time. To resolve this ISI problem a technology is used name as Adaptive equalization. The Adaptive equalizer has property to make attends to unknown time-varying channel, for this they required a special type of algorithm to update their equalizer coefficients so that they can easily track the channel changes [1].

A detailed study of adaptive algorithm is a complex work.

As is described in [2]:

- The Least Mean Square (LMS) algorithm equalizer is more stable than the zero forcing equalizer.
- The normalized LMS and other LMS algorithms are robust to variability of the input signal's statistics (such as power & other statistics).
- The RLS algorithm converges faster, but it becomes more complex with the square of the number of weights $O(x^2)$, where x is a signal. This algorithm can also be unstable when the number of weights is large.

The criterion is that the mean square error (MSE) between the desired output value and the actual output value of the equalizer should minimize.

In this paper, we take LMS, NLMS, RLS and QR-RLS algorithm with 100 ensembles and 800 independent iterations of it and then simulate them in MATLAB & compare them.

For producing a comparable level of algorithm by varying the step size 'μ' for LMS & NLMS algorithm and the same value of forgetting factor by the relation (λ=1-μ) for RLS & QR-RLS algorithm. Then find the Mean Square Error (MSE) & Mean Square Error Average (MSE_av) by using MATLAB simulation and compare the results of all FIR filter algorithms and also compare the theoretical & practical aspects of it.

The simulation results show how the QR-RLS algorithm effective in performance improvement.

Than by the help of this algorithm an equalizer is designed. The equalizer is effective with improved convergence rate and reduced MSE error especially with the QAM constellations [3].

2. PRINCIPLE OF CHANNEL EQUALIZATION

The equivalent baseband model of Channel equalization system is shown in Fig. 1

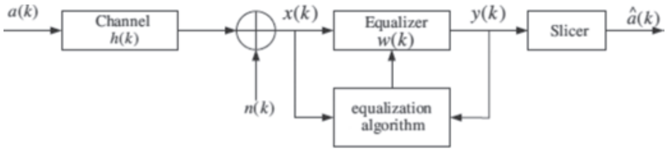

Fig 1: The Schematic Model Diagram of Channel Equalizer [3]

The received signal $x(k)$ can be written as [3]

$$x(k)=\sum_{i=0}^{L-1}h(i)a(k-i)+n(k) \tag{1}$$

where $h(k)$ is the overall complex baseband equivalent impulse response of the transmitter Filter, unknown channel and receiver Filter. L is the length of $h(k)$. $n(k)$ is taken as additive white Gaussian noise. $a(k)$ is the input data sequence which is assumed to be independent and identically distributed.

The output of equalizer can be written as [4].

$$y(k)=X^T(k)W(k) \tag{2}$$

Where

$$W(k)=\left[w_0(k),w_1(k),\ldots,w_{N-1}(k)\right]^T \tag{3}$$

$$X(k)=\left[x(k),x(k-1),\ldots,x(k-N+1)\right]^T \tag{4}$$

W(*k*) is the equalizer tap weights vector, *X*(*k*) is the input vector of equalizer. The length of equalizer tap weights is *N*.

3. ADAPTIVE FILTER ALGORITHM

A. LMS Algorithm

LMS algorithm update its weights to obtain optimal performance based on the least mean square criterion and gradient-descent methods [5].

Using gradient-descent methods to acquire w_{opt} , the weight update formula is

$$w(n) = w(n-1) + f\left(\mu, u(n), e(n)\right) \tag{5}$$

and weight update function is

$$f\left(u(n), e(n), \mu\right) = \mu e(n) u^*(n) \tag{6}$$

Where μ is the step-size factor, u(n) is the vector containing the most recent N samples of the system input signal, e(n)is system output error and u*(n) is complex conjugate of u(n). by varying the values of step size we get different values of MSE.

B. NLMS Algorithm

The main drawback of the "pure" LMS algorithm is that it is sensitive to the scaling of its input x(n). This makes it very hard (if not impossible) to choose a learning rate μ that guarantees stability of the algorithm [6]. The *Normalized least mean squares filter* (NLMS) is a variant of the LMS algorithm that solves this problem by normalizing with the power of the input. The optimal learning rate is

$$\mu_{opt} = \frac{E\left[\left|y(n) - \hat{y}(n)\right|^2\right]}{E\left[\left|e(n)\right|^2\right]} \tag{7}$$

Where y(n) is output signal, $\hat{y}(n)$ is Hermitian conjugate of y(n) and e (n) is error signal.

C. RLS Algorithm

For FIR RLS adaptive filter we take 'L' which is adaptive filter length depends on the number of coefficients or taps and it must be a positive integer. 'λ' as the RLS forgetting factor. This is a scalar unit and must lie between 0 to 1. We take small positive constant δ (delta) which used to initialize the estimate of the inverse of the autocorrelation/ covariance matrix. This matrix should be initialized to a positive definite matrix.

D. QR-RLS Algorithm

In the QR-RLS algorithm, or QR decomposition-based RLS algorithm, the computation of the least-squares weight vectors is accomplished by working directly with the incoming data matrix via the QR decomposition where in standard RLS algorithm it is accomplished by working with the (time-averaged) correlation matrix of the input data in a finite-duration impulse response. Accordingly, the QR-RLS algorithm is numerically more stable than the standard RLS algorithm [7, 8].

For n=1, 2 …………, compute

$$\begin{pmatrix} \lambda^{1/2}\Phi^{1/2}(n-1) & u(n) \\ \lambda^{1/2}p^{1/2}(n-1) & d(n) \\ 0 & 1 \end{pmatrix} \Theta(n) = \begin{pmatrix} \Phi^{1/2}(n) & 0 \\ p^{H}(n) & \xi(n)\gamma^{1/2}(n) \\ u^{H}(n)\Phi^{-H/2}(n) & \gamma^{1/2}(n) \end{pmatrix} \tag{8}$$

$$\hat{w}^{H}(n) = p^{H}(n)\Phi^{-1/2}(n) \tag{9}$$

Computed $\Phi^{1/2}(n)$ & $p^{H}(n)$ are the updated block values and $\hat{w}^{H}(n)$ is the least-squares weight vector. $\Theta(n)$ is a unitary rotation that operates on the elements of the input signal vector u(n). $\xi(n)\gamma^{1/2}(n)$ is top triangular section's output & $\gamma^{1/2}(n)$ is the bottom triangular section's output[7].

4. SIMULATION ANALYSIS & RESULTS

In this section, we simulate for LMS, NLMS, RLS & QR-RLS adaptive filter algorithm for an unknown FIR filter with different values of step size (μ) and forgetting factor (λ).

Fig.2. & Fig.3. shows the MSE curves at different values of μ for LMS and NLMS respectively. Fig.4. & Fig.5. shows the MSE curves at different values of λ for RLS and QR-RLS respectively.

In Table 1 & Table 2 the MSE-avg values are defined at different μ for LMS and NLMS respectively which shows that at μ=0.1 LMS and NLMS gives lowest Mean Square Error. In Table 3 & Table 4 the MSE-avg values are defined at different λ for RLS and QR-RLS respectively which shows that at λ=0.9 RLS and QR-RLS gives lowest Mean Square Error.

A. LMS Algorithm

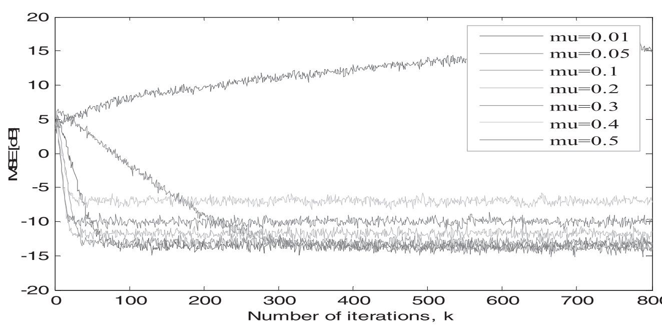

Fig 2: MSE Curve for LMS Algorithm at different mu (μ)

Table 1:
MSE_ AVG OF LMS ALGORITHM AT DIFFERENT STEP SIZE (μ)

Value of step size (μ)	Min. MSE (LMS)	Max. MSE (LMS)	MSE_AVG (LMS)
0.01	0.0311	4.872	2.45155
0.05	0.0328	3.857	1.9449
0.1	0.0722	2.6927	**1.38245**
0.2	0.0461	2.8073	1.4267
0.3	0.0347	3.5089	1.7718
0.4	0.1212	3.1569	1.63905
0.5	0.5577	38.9374	19.74755

B. NLMS Algorithm

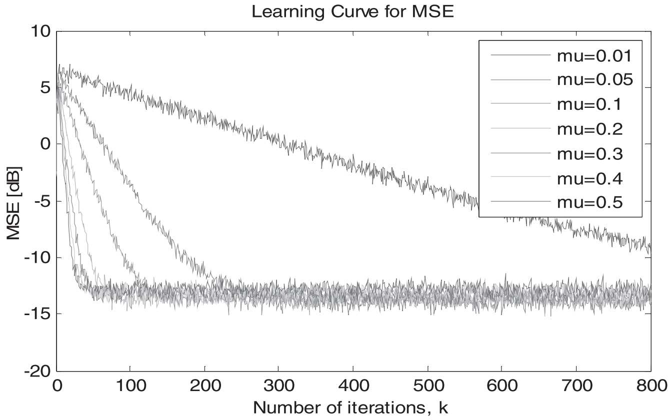

Fig 3: MSE Curve for NLMS Algorithm at different mu (μ)

Table 2:
MSE_ AVG OF NLMS ALGORITHM AT DIFFERENT STEP SIZE (μ)

Value of step size (μ)	Min. MSE (NLMS)	(NLMS)	MSE_AVG (NLMS)
0.01	0.1075	4.8075	2.4575
0.05	0.0311	4.5051	2.2681
0.1	0.035	3.1605	**1.52775**
0.2	0.0302	3.5607	1.79545
0.3	0.0316	3.8599	1.94575
0.4	0.0354	3.0472	1.5413
0.5	0.0373	3.4655	1.7514

C. RLS Algorithm

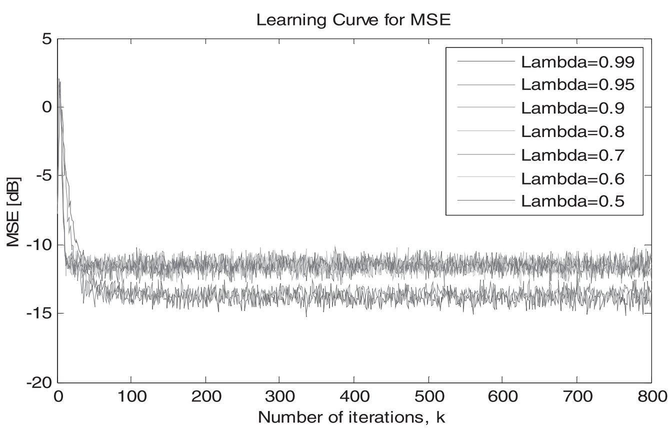

Fig 4: MSE Curve for RLS Algorithm at different lambda (λ)

Table 3: MSE_ AVG OF RLS ALGORITHM AT DIFFERENT FORGETTING FACTOR (λ)

Value of lambda (λ)	Min. MSE (RLS)	Max. MSE (RLS)	MSE_AVG (RLS)
0.99	0.0287	1.5191	0.7739
0.95	0.0284	1.4342	0.7313
0.9	0.0522	1.2882	**0.6702**
0.8	0.0469	1.3996	0.72325
0.7	0.0511	1.4138	0.73245
0.6	0.0286	1.3186	0.6736
0.5	0.0506	1.3386	0.6946

B. QR-RLS Algorithm

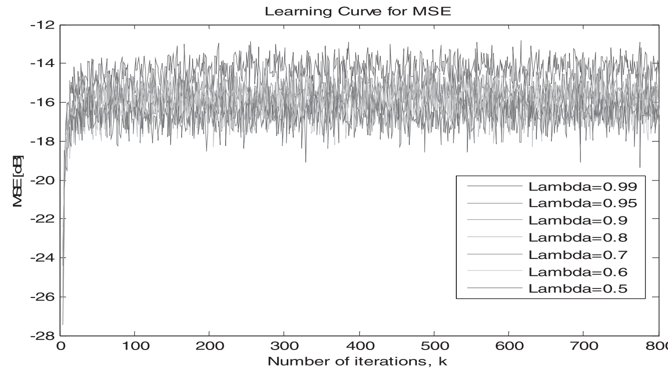

Fig 5: MSE Curve for QR-RLS Algorithm at different lambda (λ)

Table 4:
MSE_ AVG OF QR-RLS ALGORITHM AT DIFFERENT LAMBDA (λ)

Value of lambda (λ)	Min. MSE (RLS)	Max. MSE (RLS)	MSE_AVG (RLS)
0.99	0.0287	1.5191	0.7739
0.95	0.0284	1.4342	0.7313
0.9	0.0522	1.2882	**0.6702**
0.8	0.0469	1.3996	0.72325
0.7	0.0511	1.4138	0.73245
0.6	0.0286	1.3186	0.6736
0.5	0.0506	1.3386	0.6946

In Fig.6 simulation of LMS, NLMS, RLS & QR-RLS adaptive filter algorithm with the signal x is showed which is chosen according to the QAM constellation. The variation of x is normalized to 1. The complex MSK data is generated for 100 ensembles with 800 iterations (k). From the plot it is clear that with faster initial convergence speed QR-RLS has lowest MSE-av(dB) with compare to other adaptive algorithms.

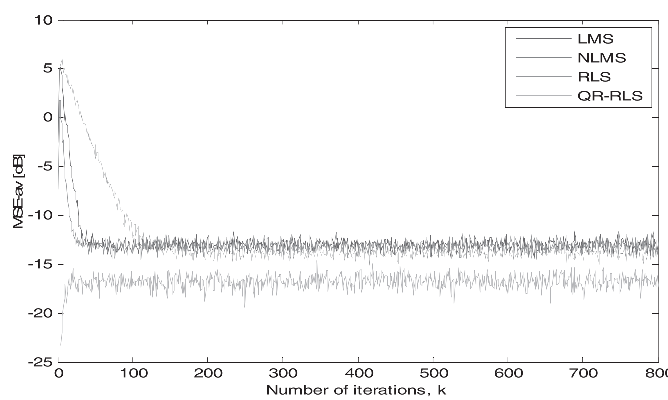

Fig 6: Comparison of MSE Curve for LMS, NLMS, RLS & QR-RLS Algorithm

Table 5:
EXPERIMENTAL RESULTS OF MSE AT DIFFERENT ALGORITHM

Algorithm	Step Size(μ)/ Forgetting Factor(λ)	No. of Filter Coefficients	No. of Iterations	MSE (dB)
LMS	μ=0.1	100	800	1.3825
NLMS	μ=0.1	100	800	1.5275
RLS	λ=0.9	100	800	0.6702
QR_RLS	λ=0.9	100	800	**0.0173**

According to Table 5 QR-RLS gives lowest MSE value compare to other algorithm at their best suitable step size (μ) or forgetting factor (λ) value. Than we use the QR-RLS algorithm for Channel Equalization. A QAM signal mixed with noise/ Error signal is received and then it feed to channel equalizer. In Fig.7 & Fig.8 comparison of Desired Signal, Received Signal and Error Signal with in Phase Components & Quadrature components is shown respectively. After passing through equalizer the scattered plot of received signal get equalized. A comparison is shown in Fig.9 between (a) received signal scatter plot (Input for Equalizer) and (b) Equalized Signal Plot (output of Equalizer).

For further analysis we fetch a sound signal in .WAV format and then mix a noise signal in it and then feed it in equalizer and we get the equalized output. A comparison is shown in Fig.10 between (a) Input Audio Signal (b) Error/ Noise Signal (c) Estimated Signal (d) Audio Signal After passing through Equalizer.

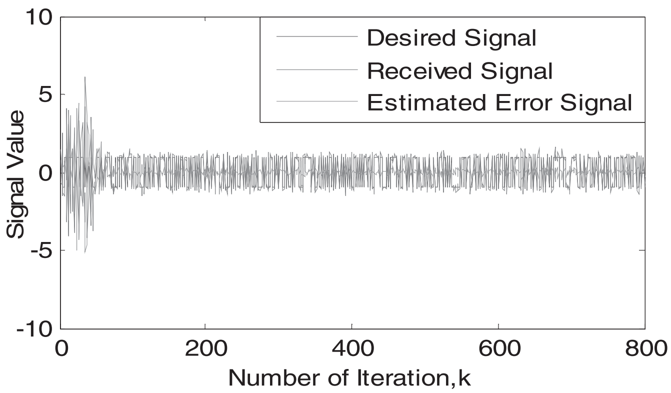

Fig 7: Comparison of Desired Signal, Received Signal and Error Signal with In Phase Components

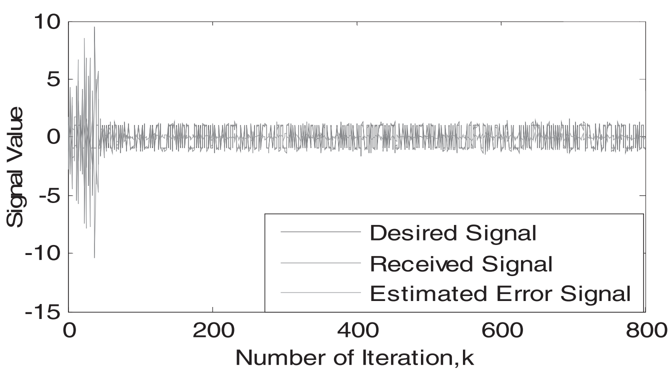

Fig 8: Comparison of Desired Signal, Received Signal and Error Signal with Quadrature Components

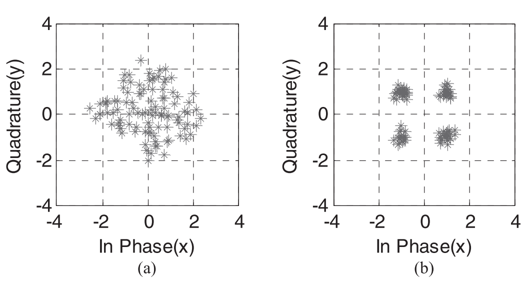

Fig 9: Comparison of (a) Received signal scattered plot (Input for Equalizer) and (b) Equalized Signal Plot (output of Equalizer) at 32-QAM

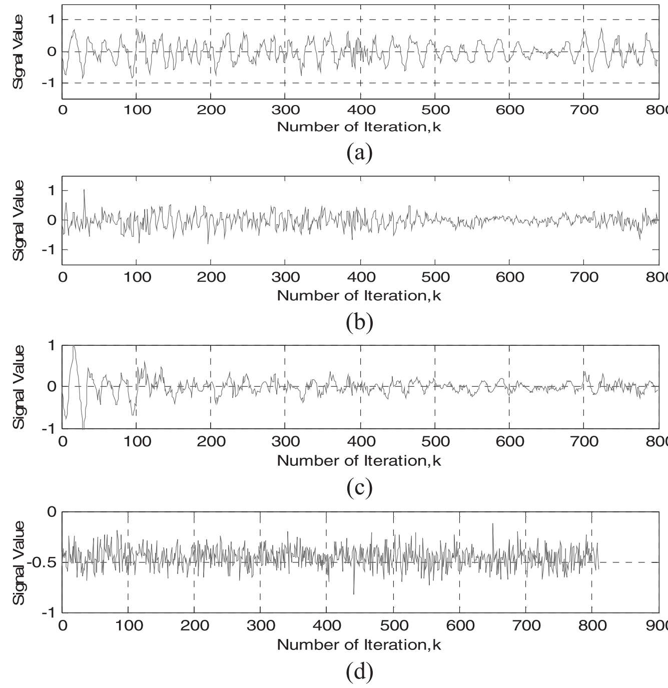

Fig 10: Analysis of an audio signal using Channel Equalizer of QRD based Algorithm (a) Input Audio Signal (b) Error/ Noise Signal (c) Estimated Signal (d) Audio Signal After passing through Equalizer

5. CONCLUSION

In this paper a channel equalizer is introduced which use advance algorithm for minimum MSE and Faster convergence rate QR-RLS adaptive algorithm with QAM modulation. QAM gives a greater bit transfer rate for the multiple carrier waves and the Equalizer will equalize the scattered plot of signal which mixed with noise signal.

Further we use some different audio signal in which we use this equalizer and equalizer gives the better performance with reduced Mean square error and faster convergence rate.

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