Inventory Model for Deteriorating Items with Inventory Dependent Demand Rate and Shortages

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Abstract: **In this paper we develop an OOQ model with inventory dependent demand rate. Shortages are allowed. Deterioration is permissible. In this model replenishment rate, ordering cost, holding cost, purchase cost, shortages cost, opportunity cost and cycle time T are considered.**

Keywords: **Deterioration, replenishment, backlogging, lead time, cycle time.**

1. INTRODUCTION

In the past research many inventory model has been developed with and without deterioration. Every item deteriorate time to time which is manufactured like food items, chemicals, Pharmaceuticals, photography, papers etc. Classical deterministic inventory models consider the demand rate to be either constant or time dependent but independent from the stock level. We have limited space for maintain the stock level. Due to low space order has to placed frequently and cost increase. If we store the items for future demand then it may be possible to damages of items due to many factors like storage conditions, weather condition, insects biting ete.

If a product is highly perishable then the cost more dependent on deterioration and ordering, the seller may prefer to backlog demand in order. But some of customers may accept backlogging during the shortage period, while the others would not. In real life smaller demand may accepted with shortages but large demand cannot be accepted.

Now if demand is dependent on inventory level, an increase in stock level for an item can attract more customers to buy it. This occurs due to visibility, popularity or variety. It means a better stock level has the positive impact on sales as well as profit.

The first attempt to describe optimal polices for deteriorating items was made by Ghare and Schrader[1], who revised form of the economic order quantity by EOQ model assuming exponential decay. Covert and Philip[2] developed some inventory model for two parameter weibull distribution for constant demand rate without shortages. Chang and Dye[3] invetigated an EOQ model for deteriorating items with time varying demand and partial backlogging. Bakdr and Urban[4] has developed a deterministic inventory system with an inventory level dependent demand rate. Teng, Cheng, and Ouyang[5] presented a model for deteriorating items with power form stock dependent demand. Hui-Ling Yanf, Jinn-Tsair Teng, Maw-sheng Chern[6] has developed a model for inventory under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Jaggi C. K. and Mittal M[7] introduced an EOQ model for deteriorating items with time dependent demand under inflationary conditions. Shah and Jaiswal[8] investigated an order level inventory for deteriorating items with a constant rate of deterioration in a deterministic environment. Singh and Shrivastava[9] introduced an EOQ model for perishable items with stock dependent selling rate and permissible delay in payment and partial backlogging. Mishra S. S. and Mishra P. P.[10] investigate price determination for an EOQ model for deteriorating items under perfect competition. Sharma and Preeti[11] developed optimum ordering interval for random deterioration with selling price and stock dependent demand rate and shortages.

In this paper, we developed an inventory model for deteriorating items with shortages, where deterioration rate depends on inventory level.

2. ASSUMPTIONS AND NOTATIONS

- Replenishment rate is infinite and lead time is zero.
- C is purchase cost per unit item.
- C_h is inventory carrying cost per unit per unit time.
- A is ordering cost.
- C_s is shortage cost per unit item.
- R unit opportunity cost of lost sales
- Deterioration rate is $\theta = \theta_0 t$
- Time t_1 for positive and zero inventory.
  T is cycle time.

$$\text{Demand } D(t) = \begin{cases} \alpha + \beta I(t), & 0 \le t \le t_1 \\ \dfrac{\alpha + e_1 I(t)}{1 + e_2(T-t)} & t_1 \le t \le T \end{cases}$$

α, β, e_1, e_2 are positive constant.

3. MATHEMATICAL MODEL

In this model inventory level id due to joint effect of deterioration and demand in interval $(0, t_1)$ and demand backlogged after this up to time T. Hence the instantaneous inventory level given by following differential equations

$$\frac{d}{dt} I(t) = -\theta . I(t) - \{\alpha + \beta\, I(t)\}, \; 0 \le t \le t_1 \tag{1}$$

$$\frac{d}{dt}I(t) = -\left\{\frac{\alpha + e_1 I(t)}{1+e_2(T-t)}\right\}, \quad t_1 \le t \le T \qquad (2)$$

Solutions of differential equations (1) and (2) under the conditions that $I(t_1) = 0$ are

$$I(t) = \alpha\Bigg[t - t_1 + \frac{\beta}{2}\left(t^2 - t_1^2\right) + \frac{\left(\beta^2+\theta\right)}{6}\left(t^3 - t_1^3\right)$$
$$- \beta t(t-t_1) - \frac{\beta^2 t^2}{2}(t-t_1) - \frac{\beta\theta}{6}\left(t^3 - t_1^3\right)$$
$$- \frac{\left(\beta^2+\theta\right)}{2}t^2(t-t_1) - \frac{\beta\theta t^2}{4}\left(t^2 - t_1^2\right)$$
$$- \frac{\beta^2\theta t^2}{6}\left(t^3 - t_1^3\right)\Bigg] \qquad (3)$$

$$I(t) = -\frac{\alpha}{e_1}\left[1 - \left\{\frac{1+e_2(T-t)}{1+e_2(T-t_1)}\right\}^{e_1/e_2}\right] \qquad (4)$$

Now

Ordering cost = A

Holding cost $= C_h \int_0^{t_1} I(t)\,dt$

$$= C_h\Bigg[-\frac{t_1^2}{2} + \beta(\theta-1)\frac{t_1^3}{6} + \left(\frac{\theta}{6} + \frac{\beta^2}{24} + \frac{\beta\theta}{8}\right)t_1^4$$
$$+ \frac{\beta^2\theta}{36}t_1^6\Bigg] \qquad (5)$$

Shortages cost $= C_s \int_{t_1}^{T} I(t)\,dt$

$$= -\frac{C_s\alpha}{e_1}\Bigg[T - t_1 + \frac{1+e_2(T-t_1)}{e_1 - e_2}$$
$$- \frac{\{1+e_2(T-t_1)\}^{-e_1/e_2}}{e_1 - e_2}\Bigg] \qquad (6)$$

Opportunity cost $= R\int_{t_1}^{T}\left[\alpha - \frac{\alpha + e_1 I(t)}{1+e_2(T-t_1)}\right]dt$

$$= R\alpha(T-t_1) + \frac{R\alpha}{e_1}\left[-1 + \{1+e_2(T-t_1)\}^{e_1/e_2}\right] \qquad (7)$$

Purchase cost $= C[I(0) - I(T)]$

$$= C\Bigg[\frac{\alpha}{e_1} + \frac{\beta\theta t_1^3}{6} - t_1 - \frac{\beta t_1^2}{2} - \left(\beta^2+\theta\right)\frac{t_1^3}{6}$$
$$- \frac{\alpha}{e_1}\{1+e_2(T-t_1)\}^{e_1/e_2}\Bigg] \qquad (8)$$

Now the total cost per cycle is given as

$TC(t_1, T) = \frac{1}{T}$ (Ordering cost + Holding cost)

+ Shortages cost + Opportunity cost + Purchase cost)

$$TC(t_1,T) = \frac{A}{T} + \frac{C_h}{T}\Bigg[-\frac{t_1^2}{2} + \beta(\theta-1)\frac{t_1^3}{6} +$$
$$\left(\frac{\theta}{6} + \frac{\beta^2}{24} + \frac{\beta\theta}{8}\right)t_1^4 + \frac{\beta^2\theta}{36}t_1^6\Bigg] - \frac{C_s\alpha}{Te_1}\Bigg[T - t_1 + \frac{1+e_2(T-t_1)}{e_1-e_2}$$
$$- \frac{\{1+e_2(T-t_1)\}^{-e_1/e_2}}{e_1-e_2}\Bigg]$$
$$+ \frac{R\alpha(T-t_1)}{T} + \frac{R\alpha}{Te_1}\left[-1 + \{1+e_2(T-t_1)\}^{e_1/e_2}\right]$$
$$+ \frac{C}{T}\Bigg[\frac{\alpha}{e_1} + \frac{\beta\theta t_1^3}{6} - t_1 - \frac{\beta t_1^2}{2} - \left(\beta^2+\theta\right)\frac{t_1^3}{6}$$
$$- \frac{\alpha}{e_1}\{1+e_2(T-t_1)\}^{e_1/e_2}\Bigg] \qquad (9)$$

To minimise the total cost $\frac{\partial}{\partial t_1}TC(t_1,T) = 0$ and

$\frac{\partial}{\partial T}TC(t_1,T) = 0$

Let t_1^* and T^* are optimum values then at these values

$\frac{\partial}{\partial t_1}TC(t_1,T) > 0$ and $\frac{\partial}{\partial T}TC(t_1,T) > 0$

4. NUMERICAL EXAMPLE:

Table & Graph 1:

A = 100 Rs, C = 10 Rs, C_h = 0.1 Rs, C_s = 2 Rs, R = 2 Rs, $t_1 = 4$, $e_1 = 0.01$, $e_2 = 1$, $\alpha = 50$, $\beta = 0.005$

θ	t	Total cost
0.5	4.64	201.099
0.52	4.37	201.106
0.54	4.15	201.112
0.56	3.84	201.121
0.58	3.56	201.131

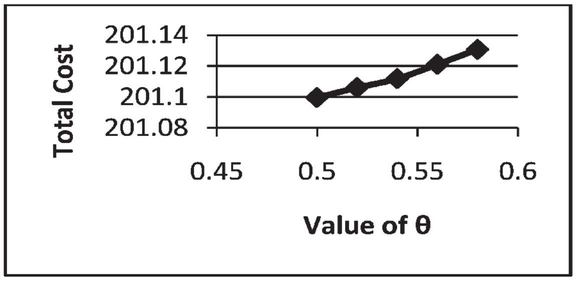


Table & Graph 2:

A = 100 Rs, C = 10 Rs, C_h = 0.1 Rs, C_s = 2 Rs, R = 2 Rs, θ = 0.5, t_1 = 4, e_2 = 1, α = 50, β = 0.005

e_1	t_1	Total cost
0.01	4.95	201.093
0.02	3.25	202.167
0.03	3.15	203.216
0.04	2.98	204.291
0.05	2.91	205.384

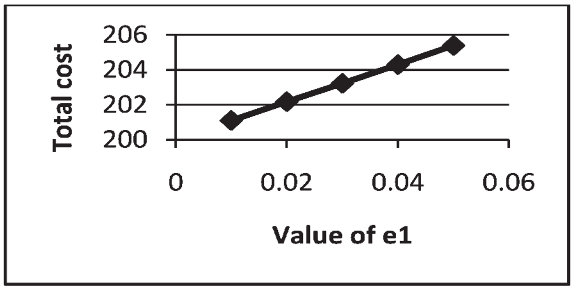


Table & Graph 3:

A = 100 Rs, C = 10 Rs, C_h = 0.1 Rs, C_s = 2 Rs, R = 2 Rs, θ = 0.5, t_1 = 4, e_1 = 0.01, α = 50, β = 0.005

e_2	t_1	Total cost
1	4.95	201.093
1.1	3.45	191.855
1.2	3.15	184.146
1.3	3.05	177.626
1.4	3.01	172.042

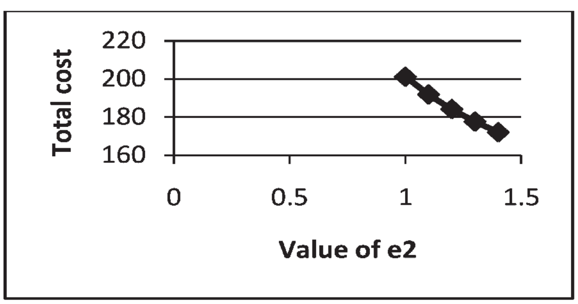


Table & Graph 4:

A = 100 Rs, C = 10 Rs, C_h = 0.1 Rs, C_s = 2 Rs, R = 2 Rs, θ = 0.5, t_1 = 4, e_1 = 0.01, e_2 = 1,β = 0.005

α	t_1	Total cost
50	4.95	201.093
45	3.43	181.022
40	3.195	160.917
35	2.956	140.811
30	2.846	120.699

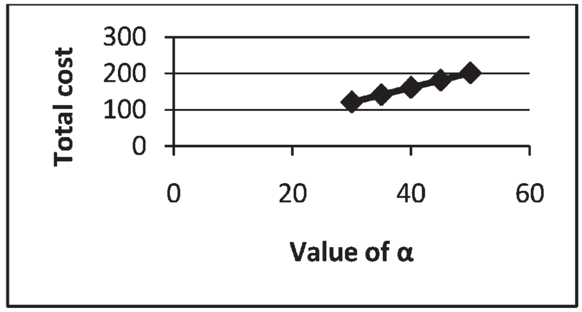
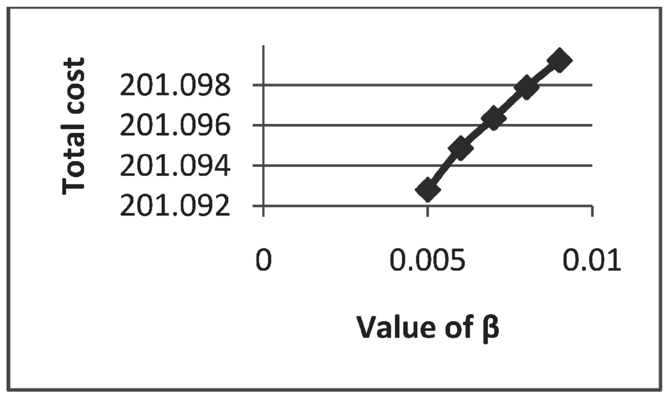


Table & Graph 5:

A = 100 Rs, C = 10 Rs, C_h = 0.1 Rs, C_s = 2 Rs, R = 2 Rs, θ = 0.5, t_1 = 4, e_1 = 0.01, e_2 = 1, α = 50

β	t_1	Total cost
0.005	4.95	201.093
0.006	4.85	201.095
0.007	4.78	201.096
0.008	4.71	201.098
0.009	4.65	201.099



5. CONCLUSION

In this paper, we developed an optimum order quantity inventory model for deteriorating items. In the model the total cost depends on different factors and the total cost increases as the parameter α, β, θ and e1 increases. Total cost decreases as e2 increases. Shortages are also allowed. In present model we use deterioration, demand, stock level, holding cost, opportunity cost and deterioration cost. In real life there are many other factors which are responsible to change the total cost. This model can extended with other effective factors and it could be done in future research

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