Free Vibration Analysis of Functionally Graded Piezoelectric Annular Plate using COMSOL® 4.2 Multiphysics Software

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Abstract: In the present age smart materials are widely used in smart mechanical structure because of its unique properties. One of the most used smart materials is functionally graded piezoelectric materials. Now a days the use of functionally graded piezoelectric materials (FGPMs) is increasing highly in making actuator and sensor and many energy harvesting technique. In this scientific paper free vibration is analyzed for FGPM annular plate. Shear effect is produced by using d15 piezoelectric coupling coefficient. Piezoelectric coupling coefficient d15 has higher value in comparison to d31 or d33. D15 piezoelectric coupling is less used for application due to complex nature of shear vibration. So to utilize d15 effect and analysis of shear induced flexural vibration a three dimensional functionally graded piezoelectric plat is used. The annular FGPM plate is polarized in radial direction and piezoelectric plate is grounded in Z-direction along the thickness. Fixed –free boundary condition is used for obtain the result. Power law is used to vary material property. COMSOL multiphysics is used for obtain the result and plot mode shapes. This result will very helpful for researcher for developing present and new actuator and sensors.

Keywords: D_{15} effect, Power law, Mode shapes, Shear-induced flexural vibration.

1. INTRODUCTION

Functionally graded piezoelectric materials (FGPMs) are used to make smart devices that are widely used in field of mechanical engineering, electrical engineering and communications. Now a days FGPM is not only used in small electromechanical systems but also as integrated structural elements. Piezoelectric actuators and sensors have novel applications for micro-electromechanical systems (MEMS) and smart material systems, especially in the medical and aerospace industries. In many such applications large deflection of piezoelectric material is required. In order to obtain the large deflection the piezoelectric materials are used in stacked form. However, due to abrupt change in properties at interface there are high interfacial stresses present in stacked actuators leading to their early failure.

Functionally graded piezoelectric materials (FGPM) have emerged as nice alternative to the stacked actuators. Due to gradual variation of material properties there are no interfacial stresses present. Functionally graded piezoelectric materials provide excellent electromechanical coupling, fast response, design flexibility. Due to such properties FGPMs are being used as promising materials for manufacturing various devices in micromechanical systems such as ultrasonic motors, actuators, micro pumps, micro valves and accelerometers etc.

Yas et al. [1] analyzed three dimensional free vibrations of functionally graded piezoelectric annular plate on elastic foundation. Li et al. [2] provided three-dimensional analysis of functionally graded piezothermoelastic annular plate. Dai et al. [3] analyzed free vibration of a functionally graded piezoelectric plate placed in a uniform magnetic field by state space method. Jiangong et al. [4] used the Legendre orthogonal polynomial series expansion method to characterize the guided waves in FGPM spherically curved plates. Wang et al. [5] investigated the axisymmetric bending of piezoelectric circular plates. Sharma et al. [10] Investigation of free vibration analysis of functionally graded annular piezoelectric plate using comsol for fixed-free boundary condition. All the literature discussed above remained confined to the d31 effect excitation of piezoceramic plates. The present paper aims at analysis of shear induced flexural vibrations of f5xed-free functionally graded piezoelectric annular plate utilizing d15 effect.

2. MODEL DESCRIPTION

The annular functionally graded piezoelectric plate (FGPM) is modeled in COMSOL Multiphysics and is shown in Fig.1. The plate is polarized along the radial direction while the electric field is applied along the thickness direction causing shear induced vibrations of the plate due to d15 effect. The typical Dimensions of the annular plate are outer diameter D=24mm, inner diameter d= 2mm and thickness h=1mm.

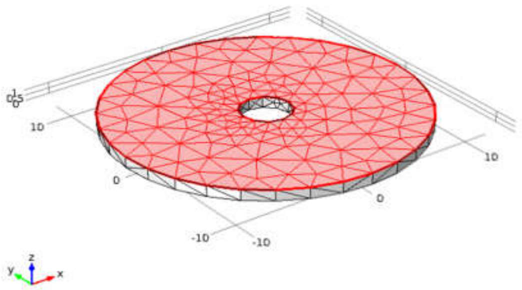

Figure 1. FGPM annular plate (D=24mm, d= 2mm, h=1mm).

The top surface of the plate is made up of Piezo material lead zirconate titanate-4 (PZT-4) while the bottom surface is made of piezo material lead zirconate titanate-5H (PZT-5H), in between along the thickness material properties vary according to the power law. The material properties can be referred from [6].

3. VALIDATION

For the FGPM annular plate excited using shear effect no such results are available in literature. For this reason the natural frequency of monolithic piezoceramic annular plate has been evaluated and compared with the results of Parashar et al. [7]. Parashar et al. [7] obtained the natural frequencies and mode shapes for fixed-free piezoceramic annular plate with the dimensions of 24 mm outer diameter, 4 mm inner diameter and 3 mm thickness. Annular plate is made of PIC 255 piezoceramic material. Comparative study of the results obtained in [7] and the present work is shown in Table 1. Herein's' represents the nodal circle and 'n' represents the nodal diameter and N represent to power law index. It can be seen from the table that finite element model made in COMSOL 4.2 Multiphysics® gives fairly accurate results.

.**Table 1:** Comparison of natural frequencies for a fixed-free annular piezoceramic plate (D=24 mm, d=4 mm and h=3mm)

S. No.	(n, s)	COMSOL® 4.2 Multiphysics	Parashar et al. [7] results
1	(1,1)	10.253	10.187
2	(2,1)	14.576	14.732
3	(3,1)	29.024	29.748
4	(4,1)	47.719	48.512

4. CONSTITUTIVE EQUATIONS

The constitutive equations to forecast the mechanical and electrical behavior of piezoelectric material can be given as [52]

$$\{S\} = [s^E]\ \{\sigma\} - [d]^T \{E\} \qquad (1)$$

$$\{D\} = [d]\ \{\sigma\} + [\varepsilon^\sigma]\ \{E\} \qquad (2)$$

Where σ the stress in (N/m2) and S is is strain.ε^σ is the electric permittivity at constant stress in (F/m).E is the electric field. $\{E\} = \{E_X\ \ E_Y\ \ E_Z\}^T$ in (V/m) and D is the elastic displacement vector $\{D\} = \{D_X\ \ D_Y\ \ D_Z\}^T$ and unit is (C/m^2). While [d] are piezoelectric coupling coefficients.
Elastic constant matrix is

$$S^E = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

Piezoelectric coupling constant matrix is

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

Permittivity (dielectric constants) matrix is given by

$$\varepsilon^\sigma = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{12} & 0 \\ 0 & 0 & \varepsilon_{13} \end{bmatrix}$$

Table 2 shows the mechanical and electrical properties of some piezoelectric materials which are used in the following chapters [8].

Table 2: Mechanical and electrical properties of some common piezoelectric materials

	PZT-4	PZT-5H
Elastic constants (10^{11} Pa)		
C_{11}	1.38999	1.27205
C_{12}	0.77836	0.80212
C_{22}	1.38999	1.27205
C_{13}	0.74283	0.84670
C_{23}	0.74283	0.84670
C_{33}	1.15412	1.17436
C_{44}	0.25641	0.22988
C_{55}	0.25641	0.22988

C_{66}	0.30581	0.23474
Piezoelectric constants (C/m^2)		
d_{15}	12.7179	17.0345
d_{24}	12.7179	17.0345
d_{31}	-5.20279	-6.62281
d_{32}	-5.20279	-6.62281
d_{33}	15.0804	23.2403
Relative permittivity ($\varepsilon^{\sigma}/\varepsilon^{0}$		
$\varepsilon_{11}/\varepsilon_0$	762.5	1704.4
$\varepsilon_{22}/\varepsilon_0$	762.5	1704.4
$\varepsilon_{33}/\varepsilon_0$	663.2	1433.6
Densityρ (kg/m^3)	7500	7500

4.1 **Power law**

To study the variation of properties in FGMs a model must be created along which mathematical function of composition can be described. V_c is the volume fraction which describe the volume of ceramic along the thickness t at any point z according to the volume index N which controls the shape of function V_c and is given by [9]

$$V_C = (z/t + 1/2)^N$$

If the total volume fraction is one and volume fraction of the metal is V_m in the FGMs is $(1-V_c)$. The power law function is written as

$$P(Z) = (P_t - P_b)V_C + P_b$$

P_t and P_b are the property of the metal at the top and bottom surfaces, N is the volume fraction index which ranges from $0<N<\infty$. At N=0, P (z) = P_t that means material is pure ceramic and if it is $N = \infty$ than material is pure metal.

4. RESULT AND ANALYSIS

In the present section natural frequency of FGPM annular plate for fixed-free boundary condition is obtained and also the effect of variation of power law index, and geometrical parameters on natural frequencies are observed here.

4.1 Effect of Inner and Outer Diameter Ratio

Table 3 displays the natural frequencies obtained for FGPM annular plate and effect of variation of inner and outer diameter on natural frequencies of annular FGPM plate. N denotes the power law index value of power law index is 0.5 which is constant for the calculation. Outer surface of the plate is fixed and inner surface is free. It can be observed that the natural frequency of the FGPM annular plate increases gradually when inner and outer diameter ratio increases.

Table 3: Natural frequency (kHz) for annular FGPM plate, Outer diameter=24, N=0.5, h=1 mm.

d/D, N=0.5	(n, s) (2,0)	(n, s) (4,0)	(n, s) (0,1)	(n, s) (0,2)
6/24	14.878	18.325	33.044	53.513
8/24	19.488	22.056	34.447	53.964
10/24	25.634	27.720	37.561	55.240
12/24	35.976	36.364	43.749	58.791

4.2 Effect of Plate Thickness

Table 4 displays the natural frequencies obtained for FGPM annular plate and effect of variation of thickness on natural frequencies of annular FGPM plate. Power law index is 0.5. It is observed here that natural frequency of all the mode shapes is increased. It is very useful for smart mechanical systems because it the frequency is more than piezoelectric materials can produce more output. First number of bracket (n) denotes nodal diameter and second number of bracket denote (s) nodal circle.

Table 4. Natural frequency (kHz) for annular FGPM plate, Outer diameter=24, N=0.5, h=1 to 4 mm.

H/D, N=0.5	(n, s) (1,0)	(n, s) (2,0)	(n, s) (3,0)	(n, s) (4,0)
1/24	4.441	6.007	12.249	21.414
2/24	8.153	11.349	23.314	39.666
3/24	11.208	16.165	32.282	53.425
4/24	13.705	20.510	40.406	64.347

4.3 Effect of Power Law Index

To observe the effect of geometrical parameters, natural frequency of FGPM annular plate is obtained for the fixed-free boundary conditions but with varying power law index. Figure 2 displays the result of natural frequencies obtained by varying power law index of the plate. Herein power law index is varied from N=0.5 to 10 while other parameters are kept constant. For calculating natural frequency H=1, D=24 and d=4mm. It can be observed in figure 2 that with the increase in the power law index natural frequencies of all the modes decreases.

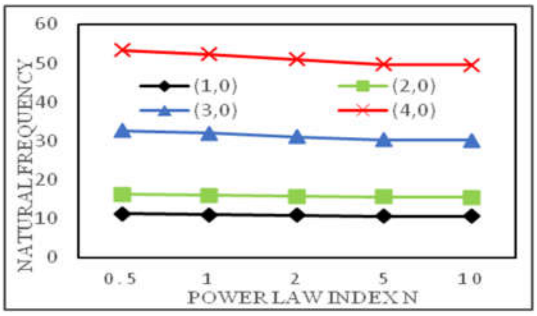

Figure 2 : Variation of natural frequency with N.

4.3 Effect of plate thickness

Figure 3 displays the result of the natural frequency for another geometrical parameter when the thickness to outer diameter ratio of the plate is increased gradually. For this the thickness of the plate h is varies from 1 mm to 4 mm while power law index is taken as 0.5. It can be observed here that there is increase in the natural frequency with increase in H/D ratio except for all the mode shapes. The increase in the frequency for all the modes may be due to the stiffness of the plate caused by increasing thickness of the plate.

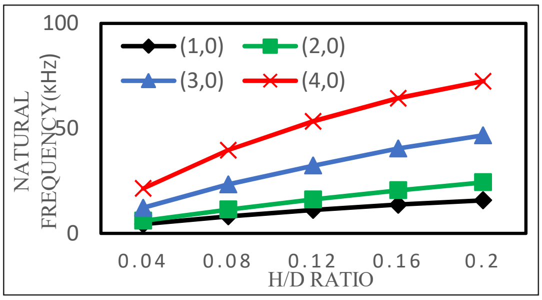

Figure 3: Variation of natural frequency with H/D

Figure 4 plot is obtained when thickness and outer diameter ratio increase. Figure 4 plot is obtained when outer diameter and inner diameter ratio of the FGPM plate increase continuously. Value of power law index is constant N=0.5 for plotting the graph. From the help of plot it is clearly notify that for fixed- free boundary condition value of natural frequency is increased in greater amount as comparative to other condition. Natural frequency for this condition increased because 4 nodal diameter is formed in very conflict area.

5 MODE SHAPE FOR FIXED- FREE ANNULAR PLATE

To analyze the free vibration of the FGPM annular plate mode shapes are plotted. Figure 5 displays the mode plot of fixed-free FGPM annular plate with dimension D=24 mm, d=12 mm, h=3 mm. The power law index is N=0.5 for this particular case.

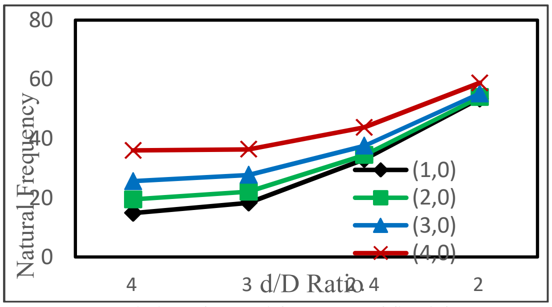

Figure 4: Variation of natural frequency of fixed-free FGPM annular plate with d/D ratio

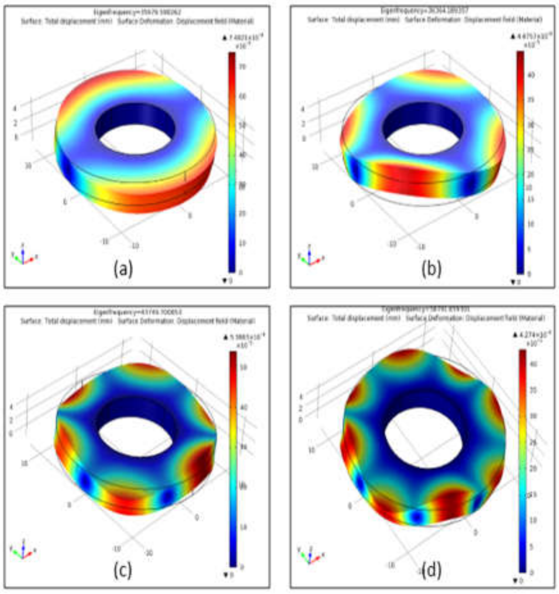

Figure 5 Mode shapes of the fixed-free annular FGPM plate (D=24 mm, d=12 mm, and h=3 mm, N=0.5) (a) (1,0) mode at 35.976 kHz, (b) (2,0) mode at 36.364 kHz (c) (3,0) mode at 43.749 kHz, and (d) (4,0) mode at 58.791.

6 CONCLUSION

In the present paper a detailed analysis of free vibrations of FGPM annular plate is presented. The modeling technique described herein should be helpful in optimizing the existing application and developing new applications based on d_{15} effect, such as ultrasonic motors and torsional actuators. Since shear induced vibrations are rarely reported in FGPM literature so there is a huge scope for future work and development. In future different geometric structures such as shell, cylinder and sphere with various loading conditions can be modeled.

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