Mechanical Static Bending Analysis of FG-Rectangular Plate Using MATLAB

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Abstract: In present work Static mechanical analysis of Fg- plate is carried out. Boundary condition for Fg-plate used is simply supported spring, which account deformation in both longitudinal as well as in transverse direction. Using above boundary condition centre plate deflection, stress & bending moment are calculated. Parametric study is also carried out in this paper. Navier solution is used for solving equation obtained from principle of virtual work. The work done in this paper is based on four variable refine plate theory which accounts both the variation of transverse shear strains across the thickness & traction free boundary condition at top and bottom surface without any shear correction factor. Material properties are graded in the thickness direction. Numerical results are obtained by a mathematical tool MATLAB.

Keyword: Navier solution, Simply supported spring, Refine plate theory, MATLAB

1. INTRODUCTION

With the advancement in technology regarding engineering, scientists and researchers are seeking for a new group of material which avoids stress concentration at interface between two layers of materials being compatible with high temperature environment, high water concentration environment and high mechanical loading environment. The conventional composites which are in light weight possessed those properties like high strength to weight and high stiffness to weight ratio are not compatible with high mechanical hygro-thermal loading environment. To overcome the drawbacks of conventional composites, FGMs were introduced in Japan in 1980. The FGM plates analysis are important from structural point of view because many real life structures are made by these plates. Structural section in trucks, ships, railways and many vehicles are made from plates. Various types of loading, different boundary conditions applying on FGM plate for real situation analysis. Buckling analysis, Bending analysis, vibration analysis and Nonlinear analysis with static and dynamic loading in mechanical, hygrothermal environment attract researchers toward FGM. For current analysis, boundary condition is used very similar to rocket launch pad.

Praveen and Reddy [1] Investigated FG ceramic-metal plate using finite element approach with varying the volume fraction according to power law . The effect of temperature field on static and dynamic analysis of plate for stress and deflection are also presented. Using the Navier's solutions for rectangular plates, based on finite element models theoretical formulation applying TSDT (third order shear deformation) are presented by Reddy [2] in 2000. The plate taken during analysis is isotropic in nature whose material property is graded by power law in thickness direction. To show the out-turn of material variation non-linear FSDT and linear TSDT results are presented. Axisymmetry formulation for FSD (first-order shear deformation) annular and circular FG plate in bending which is an extension of formulation for rectangular FG plate is carried by Reddy [3]. Solution is based on classical plate theory for deflection, force and couple resultants. By applying both mechanical and thermal load and using classical nonlinear plate theory Reddy and Cheng [4] further advances the FG plate analysis which accounted boundary and edge effect for solution of three dimensional plates. They introduce asymptotic method to find exact solution near the interior portion of plate for various plate theories. On simply supported thick FG plate small elastic deformation was studied by Vel and Batra [5].

A bending analysis, applying the mechanical, hygrothermal loading on FG plate resting on elastic foundation is carried out by A.M.Zenkour [6] in 2010. Material property, thermal coefficient, Moisture coefficient varies in thickness direction by power law. Exact solution of bending of Titanium/Zirconia plate following Navier approach is obtained. In 2014, For advance plate analysis resting on elastic foundations in thermal and moisture environment Khateeb and Zenkour [7] used refined four variable plate theory, which accounts both the variation of transverse shear strains across the thickness & traction free boundary condition at top and bottom surface without any shear correction factor.Wu Zhen [8] using the RHSDT (Reddy type higher shear deformation theory) and considering transverse normal strain studied the mechanical hygrothermal effect on FG plate. Material properties vary according to power law. Shimpi, R. P., and H. G. Patel [9] introduces two variable refined plate theory for orthotropic plate analysis. Sadah A Al Khateeb and Ashraf M Zenkour [10] has done work on refined four-unknown plate theory for advanced plates resting on elastic foundations in hygrothermal environment.

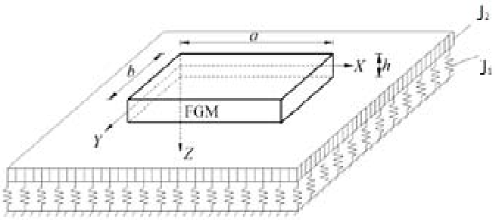

Figure 1: plate geometry and dimension

2. FOUR VARIABLE REFINE PLATE THEORY FOR RECTANGULAR PLATE

As well-known refinement is the process of smoothing the result. Similar according to the name refine plate theory gives result very accurate like higher order shear deformation theory and provides simplicity like classical plate theory. Four variable refine plate theory displacement equations are:-

2.1 Displacement equation

l, m, n are the displacement of plate in x,y and z direction respectively & a, b, h is the dimensions of the plate in x, y and z direction respectively

$$l(x,y,z)=l_0(x,y)-z\frac{\partial n_b}{\partial x}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial n_s}{\partial x} \quad (1a)$$

$$m(x,y,z)=m_0(x,y)-z\frac{\partial n_b}{\partial y}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial n_s}{\partial y} \quad (1b)$$

$$n(x,y,z)=n_b(x,y)+n_s(x,y) \quad (1c)$$

2.2 Strain equations

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{\partial l_0}{\partial x}\\ \frac{\partial m_0}{\partial y}\\ \frac{\partial l_0}{\partial y}+\frac{\partial m_0}{\partial x}\end{pmatrix}+z\begin{pmatrix}-\frac{\partial^2 n_b}{\partial x^2}\\ -\frac{\partial^2 n_b}{\partial y^2}\\ -2\frac{\partial^2 n_b}{\partial x\partial y}\end{pmatrix}+ f(z)\begin{pmatrix}-\frac{\partial^2 n_s}{\partial x^2}\\ -\frac{\partial^2 n_s}{\partial y^2}\\ -2\frac{\partial^2 n}{\partial x\partial y}\end{pmatrix} \quad (2a)$$

$$\begin{pmatrix}\gamma_{yz}\\ \gamma_{xz}\end{pmatrix}=g(z)\begin{pmatrix}\frac{\partial n_s}{\partial y}\\ \frac{\partial n_s}{\partial x}\end{pmatrix} \quad (2b)$$

2.3 Constitutive equations

Material property like Young's modulus, are vary along thickness direction only and according to power law.

$$P(z)=P_M+(P_c-P_M)\left(\frac{1}{2}+\frac{z}{h}\right)^k. \quad (3)$$

Stress strain relations are:-

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{pmatrix}=\begin{bmatrix}I_{11} & I_{12} & 0\\ I_{12} & I_{22} & 0\\ 0 & 0 & I_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix} \quad (4a)$$

$$\begin{pmatrix}\tau_{yz}\\ \tau_{zx}\end{pmatrix}=\begin{bmatrix}I_{44} & 0\\ 0 & I_{55}\end{bmatrix}\begin{pmatrix}\gamma_{yz}\\ \gamma_{zx}\end{pmatrix} \quad (4b)$$

Where $I_{11}=I_{22}=\frac{E(z)}{1-\nu^2}$

$I_{12}=\frac{\nu E(z)}{1-\nu^2}$, $I_{44}=I_{55}=Q_{66}=\frac{E(z)}{2(1-\nu)}$

2.4 Governing equations

Suppose plate is displaced infinitesimally small and principle of virtual work is applied to derive the equation of equilibrium. For current work virtual work principle leads to equation

$$\int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]d\Omega dz$$
$$-\int_{\Omega}(q-f_e)\delta n d\Omega=0 \quad (5)$$

Where Ω is the top surface & f_e is intensity of reaction.

Using equation (4a), (4b) and (5) we get

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \quad (6a)$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0 \quad (6b)$$

$$\frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial xy}+\frac{\partial^2 M_y^b}{\partial y^2}-f_e+q=0 \quad (6c)$$

$$\frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial xy}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}-f_e+q=0 \quad (6d)$$

Using these equation Zenkor [11] done his work on exponentially graded plate.

Where,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{bmatrix}$$

$$\begin{bmatrix} s_{yz}^s \\ s_{xz}^s \end{bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}$$

Also,

$$\begin{pmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{pmatrix} =$$

$$\int_{-h/2}^{h/2} I_{11}(1, z, z^2, f(z), zf(z), f^2(z)) \begin{pmatrix} 1 \\ \nu \\ \frac{1-\nu}{2} \end{pmatrix} \text{dz}$$

&

A_{22}=A_{11}, B_{22}=B_{11}, D_{22}=D_{11}, B_{22}^s=B_{11}^s, D_{22}^s=D_{11}^s, H_{22}^s=H_{11}^s

A_{44}^s=A_{55}^s=$\int_{-h/2}^{h/2} \frac{E(z)}{2(1+\nu)} [g(z)]^2 \text{dz}$

Table 1: Result for mechanical environment

k	K_0	J_0	n'	σ_x'	τ_{xy}'	τ_{xz}'
0	100	100	0.08225	0.04905850	0.0733189	-0.04120
0.5	100	100	0.0839970	0.04582558	0.0579112	-0.03338
1	100	100	0.0846885	0.04442162	0.0494819	-0.02969
2	100	100	0.0851933	0.04374115	0.0438895	-0.02631
5	100	100	0.0856012	0.04501137	0.0413165	-0.02371

2.5 Solution of equilibrium equation

After getting the equilibrium equation (6a),(6b),(6c),(6d), equation of equilibrium are solved with the help of MATLAB 2016b, a mathwork software. Navier's approach is used for equations.

In this paper we used Navier's solution so follow Navier's assumption

$$\begin{pmatrix} q \end{pmatrix} = \begin{pmatrix} q_0 \end{pmatrix} sin\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right) \quad \&$$

$$\begin{pmatrix} l_0 \\ m_0 \\ n_b \\ n_s \end{pmatrix} = \begin{pmatrix} Lcos\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right) \\ Tsin\left(\frac{\pi x}{a}\right) cos\left(\frac{\pi y}{b}\right) \\ X_b sin\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right) \\ X_s sin\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right) \end{pmatrix}$$

3. RESULT & DISCUSSION

Non-dimensional form of deflection and stress is evaluated according to following parameters:

- Center plane deflection:-

n' = 10^2*D/(a^4*q_0)*n(a/2,b/2)

- Non dimensional stress in x direction

σ_x'$_{=1/(10}$^{2} *q_0)* σ_x(a/2,b/2,h/2)

- Non dimensional shear stress in xy plane

τ_{xy}' =1/(10*$q_{0)}$ *τ_{xy}(0,0,-h/3)

- Non dimensional shear stress in xz plane

τ_{xz}' =-1/(10*$q_{0)}$ *τ_{xz}(0,b/2,0)

where non dimensional form of thickness is z' = z /h.

Also

K_0 = (a^4*k_w) /D

J_0 = (a^2*J_1) /D = (b^2*J_1) /D

D = (h^3*E_c)/12*(1- ν^2)

where Kw is the modulus of subgrade reaction and J1 , J2 are shear modulus of subgrade.

FGM plate taken in this paper consist metal at bottom surface and ceramic material at top surface. Metal taken in this paper is Aluminum and Ceramic is Zirconia. M stands for metal and C stands for ceramic in our equations.

These consists following material properties :

E_M = 70Gpa, ν = 3/10, α_M = 23∗(10−6), β_M = 0.33, K_M = 204 W/m.k, ρ_M = 2707 kg/m^3 For ceramic E_C = 151Gpa, ν = 3/10 ,α_C = 10∗(10−6), β_C = 0, K_C = 2.09 W/m.k, ρ_C = 3000 kg/m^3 where E stands for Young's modulas of elasticity, ν stands for Poission's ratio, α stands for thermal expansion coefficient, β stands for moisture expansion coefficient and ρ stands for density. Plate dimension chosen in x direction is a, in y direction is b and in z direction is h. a=200mm, b=600 mm, h=20 mm is chosen in this paper work.

From above table it can be clearly seen that by increasing the power law index, centre plane deflection also increases. Non dimensional x direction stress σ_x' decreases initially as power law index varies from 0 to 3, after that it increases. Decreasing phenomena is clearly understood by the fact that volume fraction value from bottom to top grading FGM plate is less than 1. We know that increasing the power for value less than 1, its value decreases. In our paper work for only mechanical load, Young's modulus variation follow similar pattern which directly leads to longitudinal stress.

Now ambiguity is that after the power law index greater than 3, stress value increases, this ambiguity is already explained by Nakamura and Sampath [12]. They stated that the value of power law index should be in range of (.33 to 3), as values outside from this range will produce FGM, which having too much dependency on one phase only.

Non dimensional shear stress in xy plane τ_{xy}' decreases continuously with increasing the value of power law index. Similar pattern also follow for non dimensional transverse shear stress in xz plane τ_{xz}'.

4. PARAMETRIC STUDY

Figure 2 represents dimensionless centre deflection with respect to aspect ratio b/a. Fig. 2 is plotted for changing b, keeping a constant. If reverse is the case plot will be different.

Figure3 represents dimension less centre deflection with respect to size to thickness ratio a/h.

5. CONCLUSION

The deflection is maximum for metallic surface of the plate and minimum for ceramic. As grading is shifted towards metallic composition, deflection tends to move toward increasing direction. As plate aspect ratio increases, dimensionless central plate portion deflection increases.

As thickness ratio increases ,dimensionless centre deflection decreases. With increase in power law index, plate deflection increases i.e, metallic composition deflect more than ceramic composition. If value of Winkler's and Pasternak foundation changes, deflection will also change.

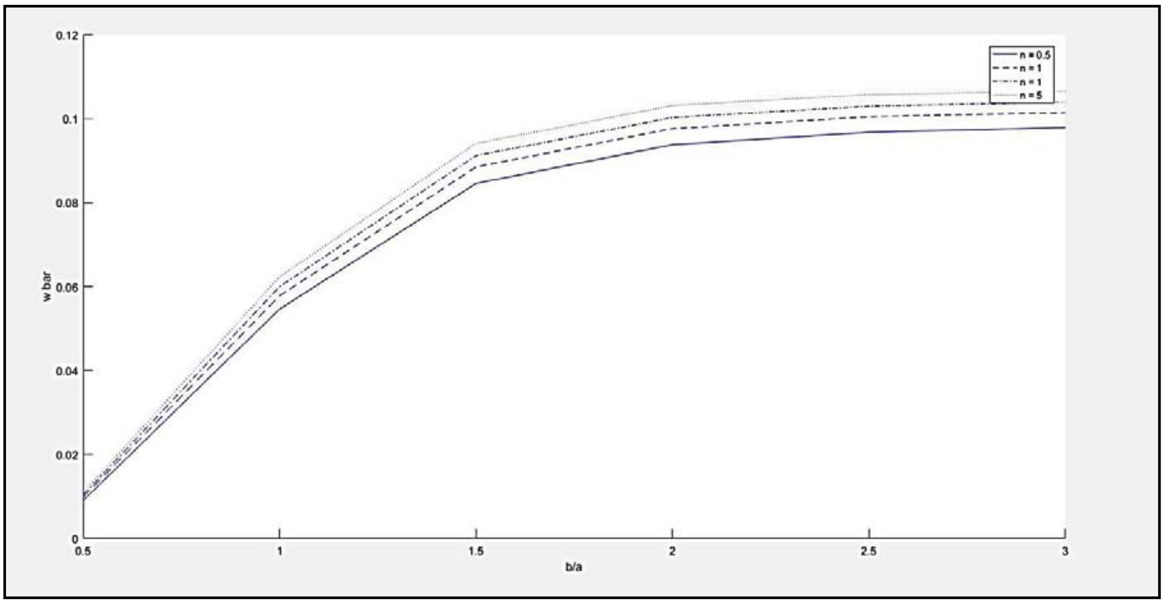

Figure 2: Dimensionless centre deflection with respect to aspect ratio b/a

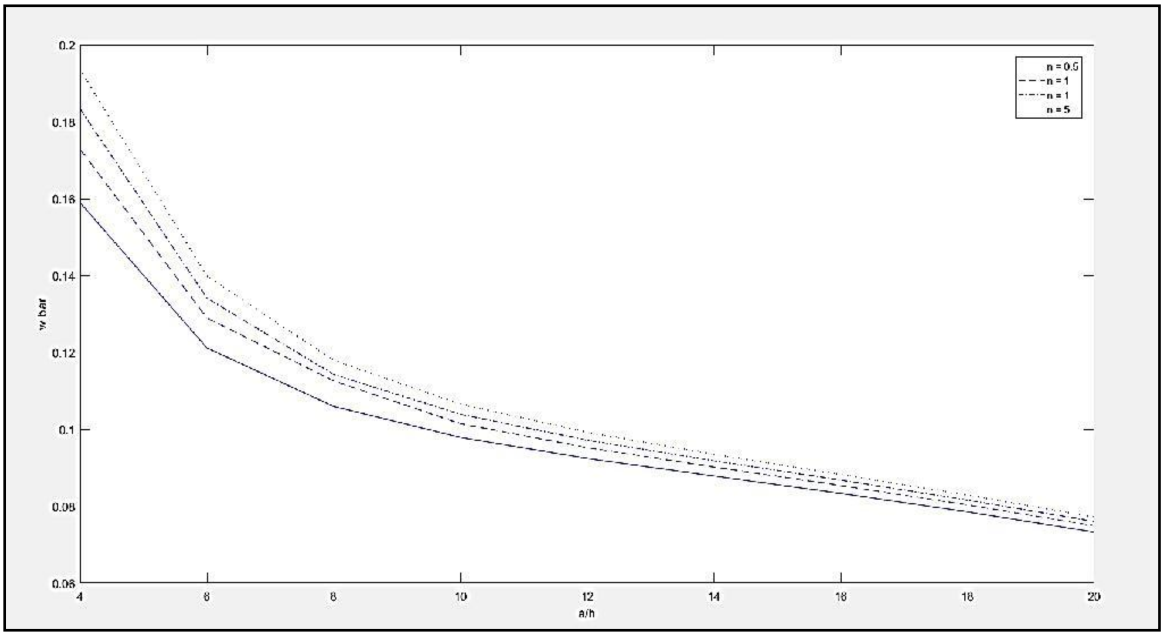

Figure 3: Dimensionless Centre deflection with respect to size to thickness ratio a/h

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