Effect of Power Law Index Variation on Eigen Frequency of Functionally Graded Piezoelectric Round Disc

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Abstract- **Piezoelectric functionally graded hollow discs are highly used materials. It makes smart devices like car hinges, drones, and microelectromechanical devices. Shear-induced eigenfrequency is much less utilized because it is very difficult to analyze. To apply functionally graded piezoelectric materials, shear vibration of the functionally graded holoow disc has been obtained by producing the d15 effect. The d15 has a higher coupling coefficient than d33 and d31, so it can be utilized to exit shear vibrations in the FGPM round hollow disc. Effect on eigenfrequency of FGPM round disk has been obtained for varying the value of power law index. The material property of the functionally graded disc varies along the thickness by helping power law. The disc is radially polarized, and an electric field is applied along the thickness.**

Keywords- Piezoelectric material, d33 effect, and power law

1. INTRODUCTION

Functionally graded materials are formed by compositing one and two materials together, and they can also be created by varying the properties of materials through the thickness direction [1-2]. This can be achieved by using power law, exponential law, etc. [3]. The vibration control study of FGPM circular disks has gained intensive attention with the extensive application of FGPM. In this study, the piezoelectric material will be very helpful in constructing sensors and actuators and making vibration control FGPM structures. Due to strong electromagnetic properties, this FGPM circular disc has gained wide engineering applications [4]. The circular disk using a hare can also detect cracks in railway wheels. This FGPM circular disk can be used in modern aeronautical, automobile, and space industries [5]. FGPM material has very good mechanical properties compared to traditional materials, such as high heat resistance, high fatigue life, and good stiffness, so it can also be used in fatigue-loading conditions [6]. Li et al. Analyzed the piezo thermoelastic coupling analysis of FGPM plates, especially for FGPM annular plates [7]. Dai et al. analyzed the free vibration of a circular plate composed of transversely isotropic FGPM for a uniform magnetic field [8]. Jiangong et al. analyzed the characteristics of guided waves in continuous FGPM spherically curved plates [9]. Wang et al. analyze the axisymmetric bending of circular plates [10]. Sharma et al. analyze the vibration behavior of FGPM annular plate by using the differential quadrature method [11].

2. MODEL DESCRIPTION

An FGPM circular disk has been selected for analysis of the effect of the power law index on eigenfrequency on the FGPM round disk. The dimensions of the plate are outer diameter DO=24 mm, the inner diameter of the circular disk is Di=2mm, and the thickness of the container is t=1mm [12]. The lower surface of the circular disk is PZT-5H rich, while the above surface is PZT-4 rich. The power law uses the make-round disk functionally graded. The round disk is radially polarized. The electric field is applied along the thickness of the round disk. Shear vibration is exited by producing the d15 effect. ‘n’ represents the power law index. 'd' represent nodal diameter, and 'c' means the nodal circle. Triangle shape elements are used to discretize annular circular disks.

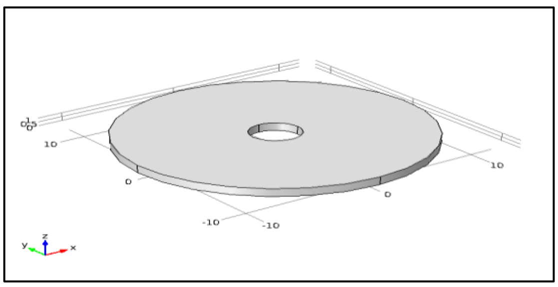

Fig. 1: Functionally graded piezoelectric circular disk (D_o=24mm, D_i= 2mm, t=1mm)

2.1 Governing equations

$$\{S\} = [s^E]\{\sigma\} - [d]^T\{E\} \quad (1)$$

$$\{D\} = [d]\{\sigma\} + [\varepsilon^\sigma]\{E\} \quad (2)$$

Where σ the stress in (N/m^2) and S is strain.$\varepsilon^\sigma$is the electric permittivity at constant stress in (F/m). E is the electric field. D is the elastic displacement vector. [d] are piezoelectric coupling coefficients [12].

3. RESULTS AND DISCUSSION

In this study, eigen frequencies of functionally graded circular disks have been evaluated for free-free, fixed-free, and fixed-fixed boundary conditions; free-free means that the inner and outer surface of the plate is free.

3.1 Analysis of Eigen frequency of round FGPM disk for measuring the effect of power-law variation for free-free boundary condition

Table 1 shows the eigenfrequency result for the free-free FGPM round disk. Here the power law index n varies from 0.5 to 100. The natural frequency of the round disk slightly decreases as the value of the power law index increases. The lowest natural frequency is obtained when the value of the power law index is 10. It can also be observed that the lowest eigenfrequency is obtained for one nodal diameter and zero nodal circles than another pattern. Hare n is the power law index, Do is the disk's outer diameter, Di is the disk's inner diameter, and t is the thickness of the plate.

Table 1 Eigenfrequency (kHz) for a free-free round FGPM disk with variation in power-law index

n	(d, c) (2,0)	(d, c) (4,0)	(d, c) (0,1)	(d, c) (0,2)
0.5	4.936	20.700	8.467	41.706
1	4.836	20.250	8.269	40.752
2	4.689	19.690	8.038	39.548
5	4.563	19.140	7.811	38.430
10	4.542	19.096	7.774	38.241
100	4.540	19.075	7.773	38.227

3.1.1 Graphical Representation of eigenfrequency when the value of power law index increases for free-free boundary conditions

Figure 2 shows the graphical representation of the result here, showing that eigenfrequency decreases significantly for zero nodal circles and two nodal diameters. While for two nodal diameters and zero nodal circles, eigenfrequency reduces by a very less amount.

3.2 Analysis of eigenfrequency of round FGPM round disk for measuring the effect of power-law variation for fixed-free boundary condition

Table 2 shows the eigenfrequency result for the fixed-free FGPM round disk. Here the power law index n varies from 0.5 to 100. The natural frequency of the round disk slightly decreases as the value of the power law index increases. The lowest natural frequency is obtained when the value of the power law index is 10. It can also be observed that the lowest eigenfrequency is obtained for one nodal diameter and zero nodal circles than another pattern.

Hare n is the power law index, Do is the disk's outer diameter, and Di is the inner diameter of the disk. And t is the thickness of the plate.

n=0.5 to 10, Do=24 mm, Di=4 mm, t=3 mm

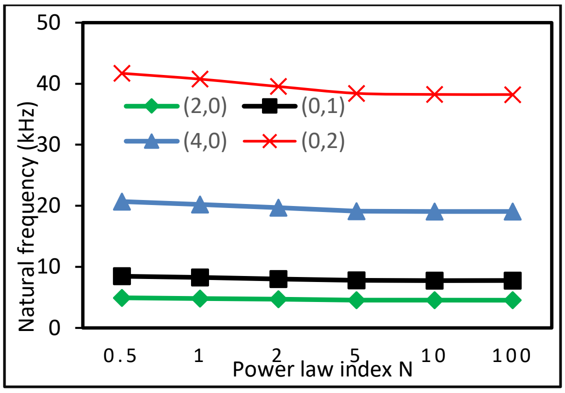

Fig. 2 : Variation of the natural frequency with power law index N for free-free functionally graded piezoelectric round disk

Table 2 Analysis of eigenfrequency of round FGPM disk for measuring the effect of power-law variation for fixed-free boundary condition

n	(d,c) (1,0)	(d, c) (2,0)	(d, c) (3,0)	(d, c) (4,0)
0.5	11.208	16.165	32.582	53.424
1	11.027	15.967	31.943	52.351
2	10.795	15.724	31.106	50.973
5	10.574	15.505	30.211	49.706
10	10.542	15.474	30.096	49.506

3.2.1 Graphical Representation of eigen frequencies when the value of power law index varies for fixed-free boundary conditions

Figure 3 shows that one nodal diameter and zero nodal circles have the lowest eigenfrequency.

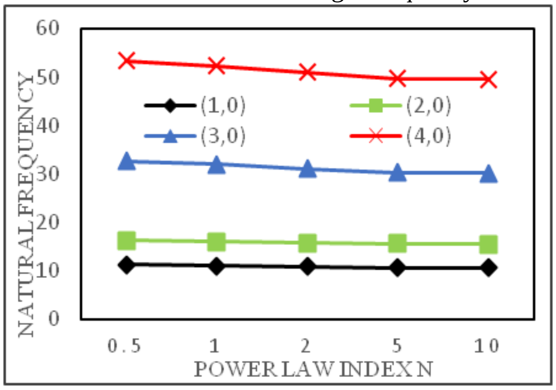

Fig. 3 : Variation of the natural frequency with power law index N for fixed-free functionally graded piezoelectric round disc

3.3.1. Graphical Representation of eigen frequencies when the value of power law index varies for fixed-fixed boundary conditions

Figure 4 represents the graphical representation of the eigenfrequency of a fixed-fixed annular plate. It can be observed here that the eigenfrequency's modes slope has a downward slope. Four nodal diameters and zero nodal circles have higher eigenfrequency than the other mode shape. It can be seen that free-free boundary conditions have the lowest eigenfrequency.

Table 3 Eigenfrequency (kHz) for a fixed-fixed round FGPM plate, outer diameter=24 mm, inner diameter=10 mm, the height of container =3 mm.

n	(0, 2)	(1, 0)	(2, 0)	(3, 0)	(4, 0)
0.5	68.154	71.267	84.217	106.44	132.68
1	66.584	69.702	82.523	104.43	130.25
2	64.616	67.741	80.387	101.87	127.16
5	62.821	65.953	78.424	99.505	124.29
10	62.566	65.699	78.144	99.164	123.87
100	62.558	65.691	78.135	99.153	123.86

4. MODE SHAPE FOR FIXED-FIXED FGPM CIRCULAR DISK

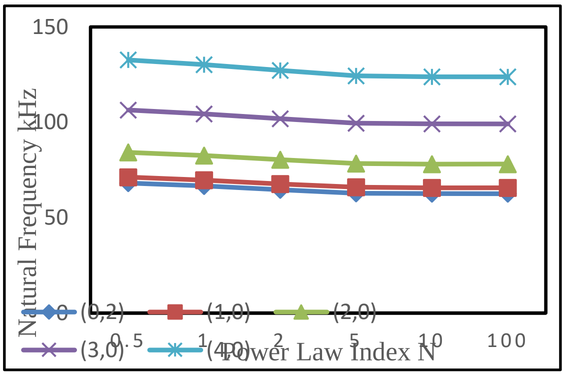

Figure 4 Variation of the natural frequency with power law index N for the fixed-fixed functionally graded piezoelectric circular disk.

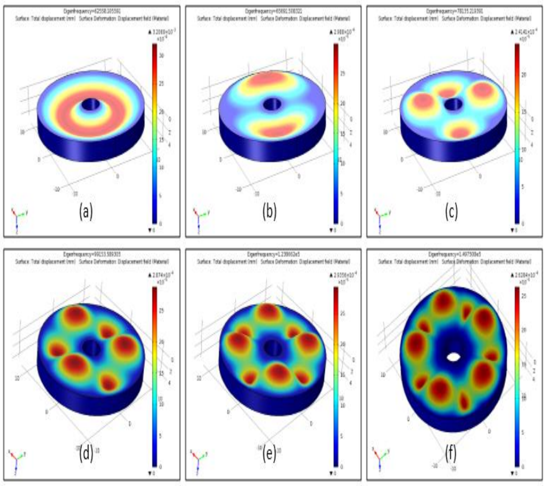

Fig. 5 : Mode shapes of the fixed-fixed annular FGPM plate (D_o=24 mm, D_i=4 mm, and t=3 mm, n=100) (a) (0, 2) mode at 62.558 kHz, (b) (1, 0) mode at 65.691 kHz (c) (2, 0) mode at 78.135 kHz, and (d) (3, 0) method at 99.153 kHz, (e) (4, 0) mode at 123.86 kHz (f) (5, 0) mode at 149Khz

5. CONCLUSION

Eigenfrequency for the FGPM plate has been evaluated, and a comparative study of different geometrical parameters has been conducted with power law variation. It is observed here that natural frequency slightly creases when the value of the power law index increases. This FGPM circular disk can be used in ultrasonic motors, drones, and many smart devices. Here, the natural frequency is higher for fixed-fixed boundary conditions than the free and fixed-free boundary conditions. The free-free plate's natural frequency is much less than the other boundary condition for the same nodal circle and nodal diameter. The natural frequency obtained here is believed to be useful for designing smart systems based on FGPM round disks by exited shear vibration. Shear-induced flexural vibration for varying power law index on the elastic foundation can be explored in future work.

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