Examining the effect of outstanding orders in stochastic inventory systems

Achin Srivastav[1],Nidhi Srivastav[2],Sunil Agrawal[3]
[1]Department of Mechanical Engineering, Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur, 302017 (INDIA)
[2]Department of Computer Science and Engineering, Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur, 302017 (INDIA)
[3]Department of Mechanical Engineering, PDPM Indian Institute of Information Technology Design and Manufacturing, Jabalpur, 482005 (INDIA)
Email- achin.srivastav@gmail.com
Received 06.04.2018 received in revised form 10.04.2018, accepted 11.05.2018

Abstract: Due to increase in the frequency of order placement and the availability of multiple suppliers at different geographical locations, the occurrences of more than one outstanding order at a single point of time have grown substantially. Conventional simultaneous approach uses lead time demand distribution which ignores outstanding orders and overestimates inventory cost. Therefore, for true estimation of inventory cost a reorder point, order quantity inventory policy is developed by recognizing the outstanding orders. The work is done in two phases. In the first phase an algorithm is designed to determine the number of outstanding orders at each period andregression equation is developed to determine the standard deviation of outstanding orders for stochastic inventory system.

In the second phase,expression for variance of shortfall distribution is developed using expected demand and variance of outstanding orders.A numerical problem is taken to illustrate the benefits of considering outstanding orders over ignoring them. Simulation is used to compute the optimal parameters of inventory system such as total inventory cost, order quantity and safety stock factor. It has been found that the use of shortfall distribution (simulation) in comparison to lead time distribution (simultaneous approach) brings down the inventory cost and order quantity by 36.12% and 72.12%. Moreover, safety stock factor is increased by 86.04% with the use of inventory shortfall distribution in place of lead time distribution. This proposed approach of shortfall distribution is widely applicable in determination of performance parameters in JIT environment, where frequent ordering leads to large number of outstanding orders at a same point of time. A novel algorithm is designed to determine the number of outstanding orders at each period. Regression equation between standard deviation of outstanding orders and time between orders is developed.The behavior of variance of outstanding orders is studied with respect to change in time between orders. The inventory shortfall distribution approach is used for reorder point order quantity inventory system.

Keywords: Algorithm, Outstanding Orders, Inventory, Stochastic.

1. INTRODUCTION

The orders, thatwere placed before and have not yet arrived are termed as outstanding orders.The stochastic lead times lead to outstanding orders as chances of order crossover increases. Order crossover is experienced when orders reaches in out of sequence to the one in which were issued (Reizebos, 2006).Globalization, rapidly changing business conditions, demanding customers and presence of global suppliers increase the chances of outstanding orders. Most of the inventory literature shows the inventory models develop considering a single outstanding order. The industries generally consider either economic order quantity or single order outstanding assumption to determine the order quantity. However, in the real-world scenario, lead times are variable and assumption of a single order outstanding is not correct. In the past, Finch (1961) first discussed on outstanding orders and gives the distribution of outstanding orders. The quantity of stock on order and the distribution of outstanding orders was determined by Zalkind (1978). Robinsonet al. (2001) extended the work of Zalkind (1978) and determined the distribution of the shortfall with an iterative algorithm. The shortfallis the difference between planned and actual inventory levels. When replenishment lead times are stochastic and independent, the outstanding orders play a crucial role in determining inventory cost. The orders are likely to cross when lead times are stochastic. Some of the significant work on order crossover are of Hadley and Whitin (1963); Anderson (1989); Navison and Burstein (1984) and Song and Zipkin (1996). Some other prominent works are of Bradley and Robinson (2005); Hayya et al. (2008) and Robinson et al. (2001) that examined order crossover in their work.Hayya et al. (2010) used distribution of effective lead time to study order crossover. Srinivasan et al. (2011) used dynamic programming to address order crossover. Recently, Bischak et al. (2014); Wensing and Kuhn (2015); Srivastav and Agrawal (2015) and Disney et al. (2016) addressed issue of order crossover in their work. In the past, researchers discussed on outstanding order issues in periodic review inventory systems. Robinson et al.(2001) considered the distribution of outstanding orders to show order crossover for complete backorder inventory model. The other works on shortfall distribution are Bradley and Robinson (2005) and Robinson and Bradley (2008). Unfortunately, none of the above work on outstanding orders has developed inventory policy for reorder point, order quantity inventory system.

This paper differs from previous works in the following manner.

1. An algorithm to find the distribution of outstanding orders for reorder point, order quantity inventory system is designed.
2. Regression equation for determining the standard deviation of outstanding orders is developed between standard deviation of outstanding orders and time between orders.
3. Shortfall distribution is determined for reorder point, order quantity inventory system.

2. NOTATIONS AND ASSUMPTIONS

2.1 Notations

The notations used in this paper are:

C_{sOO}= Total annual inventory cost assuming single outstanding orders at a point of time

C_{sF}= Total annual inventory costwhen there are more than one outstanding orders

A=ordering cost per order

D = annual demand

$E(D)$= expected value of demand

h = holding cost per unit per unit time

π= backorder cost per unit shortage

Q=order quantity

Q^*_{soo}=optimal order quantity under the assumption ofa single order outstanding

Q^*_{SF} = optimal order quantity with more than one outstanding orders

σ_{LT} = lead time standard deviation

σ_{oo} =outstanding orders standard deviation

σ_x = demand during lead time standard deviation

$\sigma_x = D\sigma_{LT}$demand during lead timestandard deviation under the single outstanding order

$\sigma_{SF} = D\sigma_{oo}$shortfall distributionstandard deviation

$\sigma_{OO=}$outstanding ordersstandard deviation

$Var(LTD)$= variance of lead time demand when assuming single order outstanding

$Var(SF)$= variance of shortfall distribution

z_0 = safety stock factor, with z~ N(0,1)

SS= safety stock

$G(z_0)$=$\int_{z0}^{\infty}(z-z_0)f(z)dz$, Expected shortage per replenishment cycle on the standard normal curve

$f(z_0)$=$\int_{z_0}^{\infty} zf(z)d(z)$

$P(z > z_0)$= stockout probability

a, b= regression coefficients in static stochastic inventory model

s= reorder point

t = time between issue of orders

t_p = prescribedtime between issue of orders

$EOQ = \sqrt{\frac{2AD}{h}}$, economic order quantity

2.2Assumptions

1. Inventory model is developed for a single item.
2. Lead times are stochastic and considered as independent and identically distributed random variable.
3. Lead times are assumed to follow exponential distribution. Exponential distribution is chosen as it is a single parameter distribution having a long heavy tail.
4. Single retailer and multiple suppliers are considered.
5. Orders of same quantity are placed at same interval of time.

3. TITLE, AUTHORS, BODY PARAGRAPHS, SECTIONS HEADINGS AND REFERENCES

3.1 Traditional approach to calculate inventory cost

Silver et al. (1998) give total approximate cost equation for complete backorders. The equation use to calculate inventory cost. The equation calculates inventory cost assuming single outstanding order as follows:

$$C_{sOO} = \frac{AD}{Q} + h\left(\frac{Q}{2} + z_0\sigma_x\right) + \frac{D}{Q}\pi\sigma_x\ G(z_0) \quad (1)$$

Here,$\sigma_x = D\sigma_{LT}$ (2)

Lot size is computed, by differentiating eq.(1) w.r.t Q, we find

$$Q^*_{sOO} = EOQ\sqrt{\left(1+\frac{\pi\sigma_x G(z_0)}{A}\right)} \quad (3)$$

Similarly, differentiating eq.(1) w.r.t. $z_0$results in the equation of tail probability.

$$P(z > z_0) = \frac{hQ}{D\pi} \quad (4)$$

3.2 Proposed simulation based approach to calculate inventory cost

Inventory model is developed for a single item, single retailer multiple supplier supply chain experiencing more than one outstanding order. To determine the outstanding orders for a reorder point, lot size inventory system a novel algorithm is proposed. The orders are placed at a fixed interval of time.

3.3 Algorithm to determine the number of outstanding orders

Step-1 Prescribe the scheduling period (time between orders), t_p.

Step-2 Trigger the orders at t_i, where t_i is instant of time at which i^{th} order is placed.

Step-3 Place the subsequent orders at ti+1, where ti+1 = ti+ tp, where i= 1....n-1.

Step-4 Generate the lead time LTi of n orders, through Rand ( ) according to distribution desiredwhere i = 1 to n.

Step-5 Obtain arrival times of ith order as ATi= ti + LTi for i= 1to n.

Step-6 Initially Outstanding Orders=0 thus list of Outstanding Order (LO) are empty.

Step-7 Assign Count=0.

Step-8 To obtain the number of Outstanding Orders at time t_i for i= 1to n, do as follows

- Add Arrival Time of order placed at time t_i i.e. AT to LO.
  Count=Count +1
- For each member 'AT' (Arrival Time) in LO
  If 'AT'<t_i then remove, 'AT' from LO and Count=Count-1.
- Number of Outstanding Orders= Count.

The newly proposed algorithm is used to determine the number of outstanding orders. Further, the standard deviation and variance of outstanding orders can be computed. The Table 1 shows that outstanding orders depend on time between issue of orders. Inventory model is developed for a single item, single retailer multiple supplier supply chain experiencing more than one outstanding order. To determine the outstanding orders for a reorder point, lot size inventory system a novel algorithm is proposed. The orders are placed at same interval of time.

Table 1: Outstanding orders in static stochastic inventory model

Period No. and Order No.	Ordering Moment when t is t_p=1, t_i	Lead Time (MEAN =2.5) LT_i	Arrival Time AT_i	Outstanding Orders
1	1	1.54	2.54	1
2	2	0.17	2.17	2
3	3	4.82	7.82	1
4	4	3.14	7.14	2
5	5	0.93	5.93	3
6	6	5.57	11.57	3
7	7	6.53	13.53	4
8	8	4.51	12.51	3
9	9	2.84	11.84	4
10	10	0.69	10.69	5
...	...	...	...	...
10000	10000	5.14	10005.14	2

Therefore, regression equations to estimate the standard deviationof outstanding orders can be written as below.

$$\sigma_{OO} = a + bt \quad (5)$$

The Figure 1 show that the variance of outstanding orders is extremely high when time between issue of orders is small, and variance of outstanding orders decreases with an increase in time between issue of orders. This explains therelevance of outstanding orders when time between issue of orders is small. The traditional approach for estimating the inventory cost works well when time between issue of orders is large as the variance of outstanding orders is negligible.However, in today's scenario where orders are placed frequently like JIT, the variance of outstanding orders play a significant role. Therefore, to ignore variance of outstanding orders will result in erroneous computation of inventory cost. Thus, it is emphasized to calculate optimal lot size by considering the variance of shortfall i.e., demand during outstanding orders.

Figure 1: Change in variance of Outstanding Orders with respect to change in time between orders

$$Var(SF) = [E(D)]^2 Var(OO) \quad (6)$$

Orders are usually placed on the basis of inventory position.Inventory position is the determined as summation of on hand inventory and quantity on order.

$$IP = OH + OO \quad (7)$$

In the traditional approach of inventory cost computation, inventory position is determined as summation of on hand inventory and the order quantity which is outstanding. However, new approach considers the possibility of more than one outstanding order.

$$IP = OH + mOO \quad (8)$$

The safety stock will also reduces using new approach as, standard deviation of demand during outstanding orders will be:

$$SS = z_0 \sigma_{SF} \quad (9)$$

$$\sigma_{SF} < \sigma_x \quad (10)$$

The total inventory cost equation for shortfall considering demand during outstanding orders, can be written as below:

$$C_{SF} = \frac{AD}{Q} + h\left(\frac{Q}{2} + z_0\sigma_{SF}\right) + \frac{D}{Q}\pi\sigma_{SF}\, G(z_0) \quad (11)$$

Differentiating equation (11) with respect to Q, results in lot size as below:

$$Q^*_{SF} = EOQ\sqrt{\left(1 + \frac{\pi\sigma_{SF}\, G(z_0)}{A}\right)} \quad (12)$$

Similarly, differentiating equation (11) with respect to z_0 results in the equation of tail probability.

$$P(z > z_0) = \frac{hQ}{D\pi} \quad (13)$$

4. RESULTS AND DISCUSSIONS

4.1 Numerical example

In order to demonstrate above results a numerical problem is considered with given data: *D*=450, *A*=$35, *h* =

\$3.5, π=\$50and lead time is 2.5. Assume lead-time is exponential distributed random variable. Compute the total inventory cost with traditional and the new approach.

With the traditional approach, assuming single outstanding order, the inventory cost can be computed by using equation (3) and (4). The results as givenin Table 2. Similarly, the new inventory shortfall distribution is used to determine inventory cost and other parameters as shown in below Table3.Regression coefficients are calculated as a= 1.7850 and b=-0.5637 (R^2 =91.41%, Adj. R^2=87.12 %). Here the iteration process starts incrementing the order quantity by one unit and corresponding optimal parameters and cost are calculated. The iteration process continues until the percentage decrease in cost and safety stock factor is less than 0.1% between two successive iterations. The results show as in Table 4 that inventory cost and order quantity decrease with the new shortfall approach and service level improves.

Table 2: Computation of cost using lead time demand distribution (simultaneous approach)

Iteration	Q	P$(z > z_0)$	z_0	$G(z_0)$	C_{soo}
1	94.86833	0.014757	2.18	0.005223	10295.78
2	290.771	0.045231	1.69	0.018604	8848.57
3	527.3409	0.082031	1.39	0.037357	8225.04
4	741.1735	0.115294	1.20	0.056235	7959.31
-	-	-	-	-	-
16	1251.973	0.194751	0.86	0.107908	7773.52
17	1252.92	0.194899	0.86	0.108012	7773.52

Table 4: Comparison of results of total annual inventory cost, lot size and safety factor with lead timedemand distribution (simultaneous approach)and a new inventory shortfall distribution (simulation).

Approach	Total Annual Inventory Cost (in \$)	Lot size	Safety factor
Lead time demand distribution (simultaneous approach)	7773.52	1252	0.86
Proposed shortfall distribution (simulation)	4961.15	349	1.60
Percentage change	-36.18%	-72.12%	86.04%

Table 3: Estimation of inventory cost, order quantity and safety stock factor using inventory shortfall approach (simulation)

Q	T	$P(z > z_0)$	z_0	% decrease in (z_0)	$G(z_0)$	σ_{SF}	σ_{SF}G(z0)	C_{SF}	% decrease inC_{SF}
1	0.0022	0.000156	3.60	-	.0000382	802.6863	11072.08833	26571.77	-
2	0.0044	0.000311	3.42	-5.11	.0000796	802.1226	23052.3082	18203.27	-31.49
3	0.0066	0.000467	3.31	-3.27	0.000123	801.5589	35438.64219	15277.86	-16.07
4	0.0089	0.000622	3.23	-2.46	0.000167	800.9952	48102.43066	13746.19	-10.03
-	-	-	-	-	-	-	-	-	-
346	0.7689	0.053822	1.61	-0.09	0.02276	608.2098	3788740.71	4976.07	-0.10
347	0.7711	0.053978	1.61	-0.09	0.022837	607.6461	3794446.09	4971.09	-0.10
348	0.7733	0.054133	1.61	-0.09	0.022913	607.0824	3800118.41	4966.11	-0.10
349	0.7756	0.054289	1.60	-0.09	0.02299	606.5187	3805757.69	4961.15	-0.09
350	0.777778	0.054444	1.60	-0.09	0.023067	605.955	3811363.93	4956.20	-0.09

5. CONCLUSION

In this paper, a new algorithm for calculation of outstanding orders for reorder point order quantity inventory system is proposed. The change in variance of outstanding orders with a change in time between orders is studied. The distribution of inventory shortfall is obtained through demand occur during outstanding orders.An inventory problem is considered to demonstrate the results. Simultaneous approach is used to determine optimal parameters for traditional stochastic inventory system (which ignores more than one outstanding order). In traditional inventory system, inventory cost is calculated using the variance of lead time distribution. On the other hand, simulation is used for estimation of optimal parameters for static stochastic inventory system experiencing more than one outstanding orders at a point of time.The results show on recognizing outstanding orders in thestochastic inventory system with respect to assumption of single outstanding order there is a remarkable reduction in both inventory cost (36.12%)and order quantity (72.12%) with a significant increase in safety stock factor (86.04%). This suggests that shortfall distribution has to be used in real world inventory systems for determination of optimal lot size, safety factor and estimation of actual inventory cost.

6. ACKNOWLEDGEMENT

Authors would like to thank the SKIT Jaipur and PDPM IIITDM for providing the support and encouragement during the research.

REFERENCES

[1] Anderson EJ, "A note on the dynamic lot-size modelwith uncertainty in demand and supply processes". Management Sci. (1989), 35, 635–640.

[2] Bischak, D.P., Robb, D.J., Silver, E.A., Blackburn, J.D., et al. (2014). Analysis and management of periodic review, order-up-to level inventory systems with order crossover. Production and Operations Management. 23(5), 762–772, doi:10.1111/poms.12072.

[3] Bradley J, Robinson L , "Improved base stock approximations for independent stochastic lead times with order crossover". Manufacturing Services and Operations Management. (2005) ,7, .319-329.
[4] Disney, S., Maltz, A., Wang, X. and Warburton, R. (2016). Inventory management for stochastic lead times with order crossovers. European Journal of Operational Research, 248(2), pp.473-486.
[5] Finch PD "The order cycle system of stock control.Journal of the Australian Mathematical Society".(1961) , 2, 209-216.
[6] Hadley G, WhitinTM , "Analysis of inventory systems". (1963), Prentice Hall, New Jersey.
[7] Hayya JC, BagchiU,Kim JG, Sun D , "On static stochastic order crossover". International Journal of Production Research. (2008), 48, 4483–4499.
[8] Nevison C, Burstein M, "The dynamic lot size model with stochastic lead times" .Management Sci. (1984) , 30,100-109.
[9] Riezebos J, "Inventory order crossovers." International Journal of Production Economics. (2006) , 104, 666–675.
[10] Robinson, L.W., and Bradley, J.R., "Further improvements on base stock approximations for independent stochastic lead times with order crossover", Manufacturing and Service Operations Management, (2008) , 10, 325-327.
[11] Robinson LW, Bradley JR, Thomas LJ, "Consequences of order crossover under order-up-to inventory policies." Manufacturing Services and Operations Management, (2001), 3, 175–188.
[12] Silver EA, Pyke DF, Peterson R "Inventory management and production planning and scheduling". (1998) , John Wiley and Sons, New York.
[13] Song JS, Zipkin PH, "The joint effect of lead time variance and lot size in a parallel processing environment", Management Science. (1996), 42, 1352–1363.
[14] Srivastav, A, & Agrawal, S. (2015). On Single Item Time Weighted Mixture Inventory Models with Independent Stochastic Lead Times. International Journal of Services and Operations Management, 22(1), 101–121.