Multiple E-K integral of I- functions Pertaining to Srivastava polynomials

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Abstract: **The multiple Erdélyi-Kober integral of the I-function, incomplete I-function connected to Srivastava polynomials is presented in this paper. Here, a few inferences using the $\overline{H}$-function, H-functions and incomplete $^{\bar{\Gamma}}I_{p,\,q}^{m,\,n}(z)$ and $^{\bar{\gamma}}I_{p,\,q}^{m,\,n}(z)$ functions are obtained. As particular examples of the original conclusion, countless more integrals emerge because of the universal nature of the I-function and Srivastava polynomials.**

Keywords: **I-function, Multiple Erdélyi - Kober integral, Srivastava Polynomials.**

1. INTRODUCTION

Rathie [1] created the I-function, an extension of the well-known $\overline{H}$-function, H-function and G-function [2]. The precise partition function of the Gaussian model in statistical mechanics, along with other helpful functions for testing hypotheses, are all contained in the I-function [1]. The following is the representation of the I-function using a Mellin-Barnes type contour integral.

$$I_{P,Q}^{M,N}[z] = I_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j, A_j; \alpha_j)_{1,N}, (a_j, A_j; \alpha_j)_{N+1,P}\\(b_j, B_j; \beta_j)_{1,M}, (b_j, B_j; \beta_j)_{M+1,Q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \theta(s) z^s\, ds \tag{1.1}$$

where

$$\theta(s) = \frac{\prod_{j=1}^{M} \Gamma^{\beta_j}(b_j - B_j s)\prod_{j=1}^{N} \Gamma^{\alpha_j}(1-a_j+A_j s)}{\prod_{j=M+1}^{Q} \Gamma^{\beta_j}(1-b_j+B_j s)\prod_{j=N+1}^{P} \Gamma^{\alpha_j}(a_j-A_j s)} \tag{1.2}$$

Also,

(i) $z \neq 0$,
(ii) $\omega = \sqrt{-1}$,
(iii) M, N, P,Q are integers satisfying $0 \le M \le Q,\ 0 \le N \le P$.
(iv) L is a suitable contour in the complex plane.
(v) An empty product is to be interpreted as unity.
(vi) A_j, j = 1,…,P; B_j, j = 1,…,Q ; α_j, j = 1,…,P and β_j, j = 1,…,Q are real positive numbers.
(vii) a_j, j = 1,…,P and b_j, j = 1,…,Q are complex numbers.

There are three different contours L of integration.

(a) L goes from $\sigma - i\infty$ to $\sigma + i\infty$, (σ is real) so that all the singularities of $\Gamma^{\beta_j}(b_j - B_j s)$, j = 1,…,M lie to the right, and all the singularities of $\Gamma^{\alpha_j}(1 - a_j + A_j s)$, j = 1,…,N lie to the left of L.
(b) L is a loop beginning and ending at $+\infty$ and encircling all the singularities of $\Gamma^{\beta_j}(b_j - B_j s)$, j = 1,…,M once in the clockwise direction, but none of the singularities of $\Gamma^{\alpha_j}(1 - a_j + A_j s)$, j =1,…,N.
(c) L is a loop beginning and ending at $-\infty$ and encircling all the singularities of $\Gamma^{\alpha_j}(1 - a_j + A_j s)$, j = 1,…,N once in the anti- clockwise direction, but none ofthe singularities of $\Gamma^{\beta_j}(b_j - B_j s)$, j = 1,…,M

In short equation (1.1) will be denoted by,

$$I_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j, A_j; \alpha_j)_{1,P}\\(b_j, B_j; \beta_j)_{1,Q}\end{matrix}\right.\right] \tag{1.3}$$

For α_j (and / orβ_j) not an integer, the poles of the gamma functions of the numerator in (1.2) are converted to branch points. The branch cuts can be chosen so that the path of integration can be distorted for each of the three contours L mentioned above as long as there is no coincidence of poles from any $\Gamma(b_j - B_j s)$, and $\Gamma(1 - a_j + A_j s)$ pair.

The sufficient conditions for convergence of (1.1) are following:

$$\theta = \sum_{j=1}^{M}|B_j\beta_j| + \sum_{j=1}^{N}|A_j\alpha_j| - \sum_{j=M+1}^{Q}|B_j\beta_j| - \sum_{j=N+1}^{P}|A_j\alpha_j| > 0 \tag{1.4}$$

And

$$|arg\, z| < \theta\frac{\pi}{2}$$

where θ is given by (1.4).

Srivastava [3] introduced the general class of polynomials

$$S_N^{\lambda}[x] = \sum_{l=0}^{[N/\lambda]}\frac{(-N)_{\lambda k}}{l!}A_{N,l}x^l, N = 0,1,2,... \tag{1.5}$$

where λ is an arbitrary positive integer and the coefficients $A_{N,l}$ (N, $l \ge 0$) are arbitrary constants, real or complex.

Let $m \ge 1$ be an integer; $\delta_i \ge 0, \gamma_i \in \Re, \beta_i > 0, i = 1,2,\dots,m$. We consider δ_i (multiorder of fractional integral); γ_i (multi-weight); β_i (additional parameter) then generalized fractional integral or multiple Erdelyi- Kober integral ([4], [5]) is defined by

$\tilde{I}f(z) = I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z) =$

$$\int_0^1 H_{m,m}^{m,0}\left[\sigma\left|\begin{matrix}(\gamma_k+\delta_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k})_{1,m}\\(\gamma_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k})_{1,m}\end{matrix}\right.\right] f(z\sigma)d\sigma;$$

if $\sum_{i=1}^{m}\delta_i > 0$ (1.6) $\tilde{I}f(z) = f(z)$ if $\delta_1 = \delta_2 = \ldots = \delta_m = 0$ (1.7)

Almost all the fractional calculus operators and most of their generalization fall in the generalized fractional calculus as special cases by taking multiplicies m= 1,2,… and special parameters.

The following lemma ([5], [6]) will be required to establish our main results

Lemma:For $\delta_i \geq 0, \gamma_i \in \Re, \beta_i > 0, (i = 1,2,\ldots,m)$,

$$p > \max_i[-\beta_i(\gamma_i+1)], \gamma_i \geq -1, p > 0$$

$$I_{(\beta_i),m}^{(\gamma_i),(\delta_i)}[z^p] = \prod_{i=1}^{m}\frac{\Gamma(\gamma_i+1+\frac{p}{\beta_i})}{\Gamma(\gamma_i+\delta_i+1+\frac{p}{\beta_i})} z^p \quad (1.8)$$

The incomplete I-functions, the generalized form of I-function described by Rathie, which are the expansion[8] formulae of well reconized Fox's H-function and many related functions. Recently, new classes of incomplete I-function ${}^{\gamma}I_{p,q}^{m,n}(z)$ and ${}^{\Gamma}I_{p,q}^{m,n}(z)$ have investigated by jangid et al.[9] and are defined as follows:

$${}^{\gamma}I_{p,q}^{m,n}(z) = {}^{\gamma}I_{p,q}^{m,n}\left[z\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_2,U_2;\lambda_2),\ldots,(u_p,U_p;\lambda_p)\\(v_1,V_1;\mu_1),\ldots,(v_q,V_q;\mu_q)\end{matrix}\right.\right]$$

$$= {}^{\gamma}I_{p,q}^{m,n}\left[z\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_j,U_j;\lambda_j)_{2,p}\\(v_j,V_j;\mu_j)_{1,q}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_L \phi(s,x)z^s\,ds, \quad (1.9)$$

and

$${}^{\Gamma}I_{p,q}^{m,n}(z) = \Gamma I_{p,q}^{m,n}\left[z\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_2,U_2;\lambda_2),\ldots,(u_p,U_p;\lambda_p)\\(v_1,V_1;\mu_1),\ldots,(v_q,V_q;\mu_q)\end{matrix}\right.\right] = \Gamma I_{p,q}^{m,n}\left[z\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_j,U_j;\lambda_j)_{2,p}\\(v_j,V_j;\mu_j)_{1,q}\end{matrix}\right.\right] =$$

$$\frac{1}{2\pi i}\int_L \psi(s,x)z^s\,ds, \quad (1.10)$$

for all $z \neq 0$where

$$\phi(s,x) = \frac{\{\gamma(1-u_1+U_1s,x)\}^{\lambda_1}\prod_{j=1}^{m}\{\Gamma(v_j-V_js)\}^{\mu_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-v_j+V_js)\}^{\mu_j}} \times \frac{\prod_{j=2}^{n}\{\Gamma(1-u_j+U_js)\}^{\lambda_j}}{\prod_{j=n+1}^{p}\{\Gamma(u_j-U_js)\}^{\lambda_j}} \quad (1.11)$$

and

$$\psi(s,x) = \frac{\{\Gamma(1-u_1+U_1s,x)\}^{\lambda_1}\prod_{j=1}^{M}\{\Gamma(v_j-V_js)\}^{\mu_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-v_j+V_js)\}^{\mu_j}} \times \frac{\prod_{j=2}^{n}\{\Gamma(1-u_j+U_js)\}^{\lambda_j}}{\prod_{j=n+1}^{P}\{\Gamma(u_j-U_js)\}^{\lambda_j}} \quad (1.12)$$

where the incomplete gamma functions $\gamma(s,\ x)$ and $\Gamma(s,\ x)$ defined as follows:

$$\gamma(s,\ x) = \int_0^x t^{s-1}e^{-t}dt \quad (R(s) > 0;\ x \geq 0), \quad (1.13)$$

and

$$\Gamma(s,\ x) = \int_x^\infty t^{s-1}e^{-t}dt \quad (x \geq 0;\ R(s) > 0\ ) \quad (1.14)$$

Known as lower and upper incomplete gamma functions respectively.

The incomplete I-functions defined in (1.9) and (1.10) exist for all $x \geq 0$under the same contour and circumstances defined by Rathie[1].

2. MAIN RESULTS

Result 1:

$$I_{(\beta_i),m}^{(\gamma_i),(\delta_i)}\Big[z^k(z+\xi)^{-\lambda}S_{N_1}^{M_1}(z^p(z+\xi)^{-q}) \times I_{P,Q}^{M,N}\left(z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(a_j,A_j;\alpha_j)_{1,P}\\(b_j,B_j;\beta_j)_{1,Q}\end{matrix}\right.\right)\Big]$$

$$= z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\times\frac{A(N_1,k_1)}{k_1!}\times$$

$$I_{P+2,Q+1+l}^{M,N+1+l}\left[\frac{z^\sigma}{\xi^\rho}\left|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(b_j,B_j;\beta_j)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(a_j,A_j;\alpha_j)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \quad (2.1)$$

Where

$\delta_i \geq 0, \gamma_i \in R, \beta_i > 0$,

$\left(\gamma_i + \frac{1}{\beta_i}(k+m+pk_i)\right) \geq -1, (i = 1,2,\ldots,l)$.

$\sigma,\rho \geq 0, l \geq 1, |arg\,z| < \frac{1}{2}A\pi$

Where

$$A = \sum_{j=1}^{M}|B_j\beta_j| + \sum_{j=1}^{N}|A_j\alpha_j| - \sum_{j=M+1}^{Q}|B_j\beta_j| - \sum_{j=N+1}^{P}|A_j\alpha_j| > 0;$$

$$Re(k) + \sigma \min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

$$Re(\lambda) + \rho \min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

Result 2:

$$I_{(\beta_i),m}^{(\gamma_i),(\delta_i)}\left[z^k(z+\xi)^{-\lambda}S_{N_1}^{M_1}(z^p(z+\xi)^{-q})\times {}^{\Gamma}I_{p,q}^{m,n}\left[z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_j,U_j;\lambda_j)_{2,p}\\(v_j,V_j;\mu_j)_{1,q}\end{matrix}\right.\right]\right.$$

$$=z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\times\frac{A(N_1,k_1)}{k_1!}\times$$

$${}^{\Gamma}I_{P+2,Q+1+l}^{M,N+1+l}\left[\frac{z^\sigma}{\xi^\rho}\left|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(v_j,V_j;\mu_j)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(u_j,U_j;\lambda_j)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \tag{2.2}$$

Where

$\delta_i\geq 0,\gamma_i\in R,\beta_i>0,$

$\left(\gamma_i+\frac{1}{\beta_i}(k+m+pk_i)\right)\geq -1,(i=1,2,\ldots,l).$

$\sigma,\rho\geq 0,l\geq 1,|arg\,z|<\frac{1}{2}A\pi$

Where

$$A=\sum_{j=1}^{M}|B_j\beta_j|+\sum_{j=1}^{N}|A_j\alpha_j|-\sum_{j=M+1}^{Q}|B_j\beta_j|-\sum_{j=N+1}^{P}|A_j\alpha_j|>0;$$

$$Re(k)+\sigma\min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

$$Re(\lambda)+\rho\min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

Result 3

$$I_{(\beta_i),m}^{(\gamma_i),(\delta_i)}\left[z^k(z+\xi)^{-\lambda}S_{N_1}^{M_1}(z^p(z+\xi)^{-q})\times {}^{\gamma}I_{p,q}^{m,n}[z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_j,U_j;\lambda_j)_{2,p}\\(v_j,V_j;\mu_j)_{1,q}\end{matrix}\right.\right]$$

$$=z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\times\frac{A(N_1,k_1)}{k_1!}\times$$

$${}^{\gamma}I_{P+2,Q+1+l}^{M,N+1+l}\left[\frac{z^\sigma}{\xi^\rho}\left|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(v_j,V_j;\mu_j)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(u_j,U_j;\lambda_j)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \tag{2.3}$$

Where

$\delta_i\geq 0,\gamma_i\in R,\beta_i>0,$

$\left(\gamma_i+\frac{1}{\beta_i}(k+m+pk_i)\right)\geq -1,(i=1,2,\ldots,l).$

$\sigma,\rho\geq 0,l\geq 1,|arg\,z|<\frac{1}{2}A\pi$

Where

$$A=\sum_{j=1}^{M}|B_j\beta_j|+\sum_{j=1}^{N}|A_j\alpha_j|-\sum_{j=M+1}^{Q}|B_j\beta_j|-\sum_{j=N+1}^{P}|A_j\alpha_j|>0;$$

$$Re(k)+\sigma\min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

$$Re(\lambda)+\rho\min_{1\leq j\leq M}\left[Re\left(\frac{b_j}{B_j}\right)\right]$$

Proof:

In order to prove (2.1), (2.2) and (2.3), we first translate the I-function into the Srivastava polynomials and the Mellin-Barnes integral (1.1) using (1.6). With a few minor simplifications, applying the Lemma (1.8) and reversing the sequence of integration and summation, we reach the desired outcome.

3. Special Cases:

(i)On taking $\alpha_j=1$ $(j=N+1,\ldots,P)$ and $\beta_j=1(j=1,\ldots,M)$in (0.3.3) the I-function reduces to the $\bar{H}$-function given by Inayat Hussain [7] as follows :

$$I_{(\beta_i),m}^{(\gamma_i),(\delta_i)}\left[z^k(z+\xi)^{-\lambda}S_{N_1}^{M_1}(z^p(z+\xi)^{-q})\times I_{P,Q}^{M,N}\left(z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(a_j,A_j;\alpha_j)_{1,N},(a_j,A_j;1)_{N+1,P}\\(b_j,B_j;1)_{1,M},(b_j,B_j;\beta_j)_{M+1,Q}\end{matrix}\right.\right)\right]$$

$$=z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\frac{A(N_1,k_1)}{k_1!}\times$$

$$\bar{H}_{P+2,Q+1+l}^{M,N+1+l}\left[\frac{z^\sigma}{\xi^\rho}\left|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(b_j,B_j)_{1,M},(b_j,B_j;\beta_j)_{M+1,Q},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(a_j,A_j;\alpha_j)_{1,N},(a_j,A_j)_{N+1,P}\\(1-\lambda-qk_1;\rho),(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \quad (3.1)$$

(ii) Taking $\alpha_j = 1\ (j = 1, \ldots, P)$ and $\beta_j = 1 (j = 1, \ldots, Q)$ the I-function reduces to the well-known Fox's H-function as

$$I^{(\gamma_i),(\delta_i)}_{(\beta_i),m}\Big[z^k(z+\xi)^{-\lambda}S^{M_1}_{N_1}(z^p(z+\xi)^{-q})\times I^{M,N}_{P,Q}\left(z^\sigma(z+\xi)^{-\rho}\middle|\begin{matrix}(a_j,A_j;1)_{1,P}\\(b_j,B_j;1)_{1,Q}\end{matrix}\right)\Big]$$

$$= z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\times\frac{A(N_1,k_1)}{k_1!}\times$$

$$H^{M,N+1+l}_{P+2,Q+1+l}\left[\frac{z^\sigma}{\xi^\rho}\middle|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(b_j,B_j)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\left.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(a_j,A_j)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \quad (3.2)$$

(iii) Taking $\lambda_j = 1\ (j = 1, \ldots, m)$ in (2.2) the *I*-function reduces to incomplete ${}^{\bar{\Gamma}}I^{m,\,n}_{p,\,q}(z)$ -function [10] as

$$I^{(\gamma_i),(\delta_i)}_{(\beta_i),m}\Big[z^k(z+\xi)^{-\lambda}S^{M_1}_{N_1}(z^p(z+\xi)^{-q})\times {}^{\Gamma}I^{m,\,n}_{p,\,q}[z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(u_1,U_1;1:x),(u_j,U_j;1)_{2,p}\\(v_j,V_j;\mu_j)_{1,q}\end{matrix}\right]$$

$$= z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\times\frac{A(N_1,k_1)}{k_1!}\times$$

$${}^{\bar{\Gamma}}I^{M,N+1+l}_{P+2,Q+1+l}\left[\frac{z^\sigma}{\xi^\rho}\middle|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(v_j,V_j;\mu_j)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\left.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(u_j,U_j;1)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \quad (3.3)$$

(iv) Taking $\mu_j = 1\ (j = n+1, \ldots, p)$ in (2.3) the *I*-function reduces to incomplete ${}^{\bar{\gamma}}I^{m,\,n}_{p,\,q}(z)$ -function[10] as

$$I^{(\gamma_i),(\delta_i)}_{(\beta_i),m}\Big[z^k(z+\xi)^{-\lambda}S^{M_1}_{N_1}(z^p(z+\xi)^{-q})\times {}^{\gamma}I^{m,\,n}_{p,\,q}[z^\sigma(z+\xi)^{-\rho}\left|\begin{matrix}(u_1,U_1;\lambda_1:x),(u_j,U_j;\lambda_j)_{2,p}\\(v_j,V_j;1)_{1,q}\end{matrix}\right]$$

$$= z^k\xi^{-\lambda}\sum_{m=0}^{\infty}\sum_{k_1=0}^{[N_1/M_1]}\frac{(-1)^m}{m!}\frac{z^{m+pk_1}}{\xi^{m+qk_1}}(-N_1)_{M_1k_1}\frac{A(N_1,k_1)}{k_1!}\times$$

$${}^{\bar{\gamma}}I^{M,N+1+l}_{P+2,Q+1+l}\left[\frac{z^\sigma}{\xi^\rho}\middle|\begin{matrix}(1-\lambda-m-qk_1;\rho),\\(v_j,V_j;1)_{1,Q},(1-\lambda-qk_1;\rho),\end{matrix}\right.\left.\begin{matrix}(-\gamma_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i}),(u_j,U_j;\lambda_j)_{1,P}\\(-\gamma_i-\delta_i-\frac{1}{\beta_i}(k+m+pk_i);\frac{\sigma}{\beta_i})\end{matrix}\right] \quad (3.4)$$

3. CONCLUSION

The results were not only quite broad in nature, but they were also presented in a concise manner, making them suitable for application. The current finding brings together many new discoveries in an intriguing way and extends them.

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