An EOQ Inventory Model for Deteriorating Items with Dependent Demand Rate under Trade Credit

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Abstract: **In the research document, I have analysed a deterministic inventory model, using a demand rate that varies over time, including deterioration of items that start after passing some particular time. Time to credit the amount is also available, for that time there is no interest to charge. But after that time highly interest rate will be applicable on remain amount. As per strategy, Vendor has some reserved money to pay initial payment but to accumulate large amount or benefit he use time of credit limit at most. First, I given a mathematical inventory model to understand behaviour of system after that I find optimal solution in different scenarios. To verification of model, there is numerical fitting in the article. Graphical presentations also show relationship between parameter and revenue.**

Keywords: **Deterioration, time dependent demand rate, Trade credit, holding cost.**

1. INTRODUCTION

In this research paper, I try to establish an economic inventory order quantity model, here I have assumed that a supplier was expected to received amount for the items as soon as the goods were delivered. As market strategy, this policy is not followed by many receivers. So, in today's commercial strategy, normally it is much seen that a vender has avail the risk of advanced supply without any payment. It is done to increase their market capturing. And vender does not apply any penalty or interest if the remaining debt is completed within the authorized time frame for settlement, after passing this time period, balance to be paid with high interest rate.

Ghare and Schrader [6] made the initial attempt to establish appropriate policies for degrading items., they introduced an EOQ inventory model assuming exponential decay of inventory items. In the past, Covert R. P. and Philip G. C. [3] developed an economic order quantity inventory model, this model has applied for deteriorating items and deterioration follow the Weibull rule, they also use constant demand rate without shortages. W. Haley and R. C. Higgin [10] have been introduced an inventory system with policy of financing system with trade credit policy. Donaldson W. A. [5] developed inventory replenishment model, in which demand follow the linear trend. He has also given analytic solutions to the prescribed model. S. K. Goyal [9] has published an article on the economic order quantity inventory model having the conditions of delay in payments up to permissible time. U. Dave [4] had his work featured on an order level inventory model including declining of goods and demand rate has the nature of variability instantaneous and items replenishment is also available with discrete nature. Goswami and Chaudhuri [7] have been explored an economic order quantity inventory system for items having deterioration with shortages and inventory demand pattern follow the linear trend. A. Goswami and K. S. Chaudhuri [8] has investigated an economic order quantity model for decling items with a linear demand trend, also they have been considered finite rate condition on replenishment, and inventory pattern allow the shortages. C. K. Jaggi and S. P. Aggarwal [13] has given an inventory model featured a scheme of credit revenue scheme in economic ordering quantity policies for declining goods. S. P. Aggarwal and C. K. Jaggi [1] have generated an inventory model to recognise ordering policies of items having deteriorating rate and there is also permissible delay time in payments. Hariga [11] has given an article on optimal economic order quantity models, this article having deterioration of items with time changing demand. Bhunia and Maiti [2] has produced a deterministic inventory model for variable production. M. Jamal et al [14] have been proposed an inventory model to establish purchasing policy in which items are of deteriorating nature, shortages are also allowed in this model and there is payment may be delay upto permissible time. N. Kumar and A. K. Sharma [16] created an optimum ordering interval model that included known requirement and items having deterioration with moving rate. This model also includes shortages. Teng, Chang, Dye and Hung [24] expressed an optimal inventory strategy with replenishment of items with changing demand, deteriorating in items. There is also partial backlogging. B. N. Mondal et al [18] have been introduced a policy of inventory system, they have deteriorating of items and the demand dependent on time. A. K. Sharma et. Al [21] has introduced an inventory model with decling items and lot size having the deterministic property, they also study the effect of payments delay time it means permissible delay item. In this system the demand follows the power pattern rule, and the deterioration is random. M. Kumar et al [15] has proposed optimal ordering inventory policy for items that prices depend on variable demand rate under trade credits. Mishra and Mishra [17] established an economic order quantity inventory model that included deteriorating items under the nature of perfect competitions, they study effect on pricing of items. Soni and Shah [23] has given a strategy optimal order quantity system, that have the demand depending of stock level and payment scheme follow the progressive rule. Singh and Shrivastava [22] established an economic order quantity inventory model for perishable goods with selling price influence by level of stock and payment delay is also permissible. For the inventory level partial backlogging is also allowed. C. K. Jaggi et al. [12] presented an inventory model, in which they use imperfact quality deteriorating items with inflation, they also permit time delay to make the payment and they study about the pricing and replenishment policies of such type of items. Sharma and Preeti [20] introduced an inventory model having the random deterioration of items. In this model demand rate is not constant that depends on stock level as well as selling price. Shortages of goods are also allowed in that model. Teng, Krommyda, Skouri and Lou [25] have represented a depth expansion of the efficient ordering policy for items that demands are stock dependent, they also study the effect of progressive payment scheme. Sarkar [19] has created an EOQ inventory model, in which payment delay was permitted and deterioration rate also varies with time. Teng, Min and Pan [26] have developed an EOQ model, where non-decreasing demand and trade credit financing is applicable. Tripathy et al (27) have published an article on an EOQ inventory model, they have been declared demand rate as constant for goods that are non-instantaneous deteriorating. For the payment by customer the follow progressive rule for financial trade credit. They have taken constant lead time and instantaneous replenishment rate without unsatisfied demand.

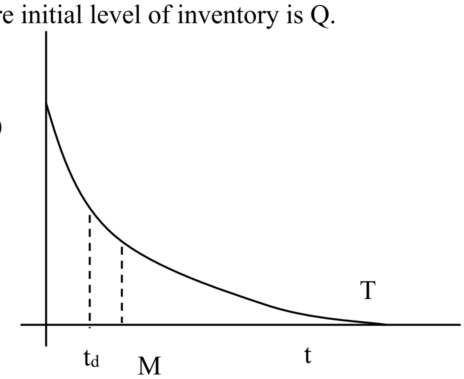
In the market strategy, demand may be fluctuated by time, stock level, pricing etc. Today scenario, the marketing policy play an important role to develop new demand. Selling price is a factor in field of business and inventory policies also. Here in the model, I tried to establish an inventory policy, at the initial time, the goods are new/fresh and have little time to best utilization i.e. no initial deterioration, this deterioration of goods/food depends on time and start after some definite time. Holding cost is constant. The demand of items depends on time i.e. it is function of time. Buyer has some credit time to make payment without any interest. But after this time, supplier will charge higher rate of interest on remaining amount. Buyer may make some initial payment to avoid fine or extra payment.

2. ASSUMPTIONS AND NOTATIONS

- $d(t) = at =$ Demand linearly depend on time.
- $I(t) =$ Instantaneous inventory level
- t_d= time of no deterioration
- α = rate of deterioration.
- There is no Shortages.
- Q = initial inventory level.
- q is order cost a cycle time.
- C_p =Inventory purchase cost of unit item.
- h = inventory holding cost of unit item.
- Payment delay is allowed and $M =$ time available without interest to settle the account.
- p selling price of unit item
- R is generated revenue in a cycle time.
- No lead time.
- T =Length of a cycle.
- Ie =earned Interest per Rs per unit time.
- Ip =paid Interest per Rs per unit time.

3. MATHEMATICAL MODEL AND SOLUTION

As per our assumption, the suitable inventory level is represented in figure. The level of instantaneous inventory $I(t)$is a function of 't' but it pursue by dual impact of time dependent demand and deterioration. By the graph we can judge, the level of inventory gradually drops and reaches to empty at time T. where initial level of inventory is Q.



The following differential equations can present the relation between inventory level, time, demand and deterioration

$$\frac{d}{dt}I_1(t) = -td, \quad 0 \le t \le t_d \qquad \ldots(1)$$

$$\frac{d}{dt}I_2(t) = -td - \alpha I(t), \qquad t_d \le t \le T \ldots(2)$$

Boundary level values are

$$I(t) = Q, \text{ at } t = 0$$
$$I(t) = 0, \text{ at } t = T$$

Now solutions of above differential equations (1) and (2) are given as

$I_1(t) = \frac{a}{2\alpha}\{\alpha(t_d^2 - t^2) + (T - t_d)(1 + \alpha T)\}$ ...(3)

$I_2(t) = \frac{a}{2\alpha}(T - t)(1 + \alpha T)$ ...(4)

And

$Q = \frac{a}{2\alpha}\{\alpha t_d^2 + (T - t_d)(1 + \alpha T)\}$ ...(5)

Order cost per cycle = q

Purchase cost = $\frac{C_p a}{2\alpha}\{\alpha t_d^2 + (T - t_d)(1 + \alpha T)\}$

Inventory holding cost is given as

HC = $\int_0^T I(t)dt$

$= \frac{ah}{12\alpha}\{4\alpha t_d^3 + 3(1 + \alpha T)(T^2 - t_d^2)\}$

Total cost is given as

TC= Ordering cost + Purchase cost + Holding cost

$= q + \frac{C_p a}{2\alpha}\{\alpha t_d^2 + (T - t_d)(1 + \alpha T)\} + \frac{ah}{12\alpha}\{4\alpha t_d^3 + 3(1 + \alpha T)(T^2 - t_d^2)\}$

There are two cases

(i) $0 \le M \le t_d$

(ii) $t_d \le M \le T$

Case (i) In this case the credit time M occurs before the time to start deterioration. Then there will be no applicable interest. And he can generate amount due to sale of goods and interest. The net revenue is amount accumulated after paying due amount. And interest will be generate on rest amout up to end to cycle time T. The retailer will gemerate revenu by selling values to end of cycle time T and he will also receive interest on this amount.

i.e.Earned interest = $\frac{Mpa}{6\alpha}\{2\alpha t_d^3 + 3t_d(T - t_d)(1 + \alpha T)\}$

Net revenu

$$R_1 = \{1 + I_e(T - M)\}\frac{a}{2\alpha}\Big[\frac{Mp\alpha}{3}\{t_d^2(2t_d + 3I_e) - I_e M^2 + (T - t_d)(1 + \alpha T)\{Mp(t_d + I_e) - C_p\}\Big] + \frac{apI_e}{12\alpha}\Big[2\alpha(2t_d^3 + M^3 - Mt_d^2) + 3(T - t_d)(1 + \alpha T)(T + t_d - 2M)\Big]$$

Now, to maximize the net revenue, we apply the optimality test.

Case (ii) The credit time M occurs after the time to start deterioration. Then there will be charged some interest or penalty. He will earn net revenue due to sale and interest on earned amount after paying due amount and penalty. The retailer will earn interest on rest amout up to end to cycle time T. The retailer will also accumulate revenu by selling items to end of cycle time T and he will also receive interest.

i.e. Charged interest = $\frac{a}{4\alpha}C_p I_p(1 + \alpha T)(M - t_d)(2T - M - t_d)$

Earned interest = $\frac{paI_e}{6\alpha}\{4\alpha t_d^3 + 3(T^2 - t_d^2)(1 + \alpha T)\}$

Net revenu

$$R_1 = \{1 + I_e(T - M)\}\frac{a}{4\alpha}\Big[\frac{2pI_e\alpha}{3}\{t_d^3 + (1 + \alpha T)(T^2 - t_d^2)\} - C_p I_p(1 + \alpha T)(M - t_d)(2T - M - t_d)\Big] + \frac{apI_e}{4\alpha}(T - M)^2$$

Now, to maximize the net revenue, we apply the optimality test.

5. NUMERICAL EXAMPLES

5.1 Case 1: $0 \le M \le t_d$

Table & Graph 1: effect of M on total revenue.

Let we take the parameters T = 1, *a* = 0.1, ∝ = 0.2, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	M	R_1
0.8	0.51	51001.26
0.8	0.52	52001.26
0.8	0.53	53001.26
0.8	0.54	54001.27
0.8	0.55	55001.27
0.8	0.56	56001.27
0.8	0.57	57001.28
0.8	0.58	58001.28
0.8	0.59	59001.28
0.8	0.6	60001.28

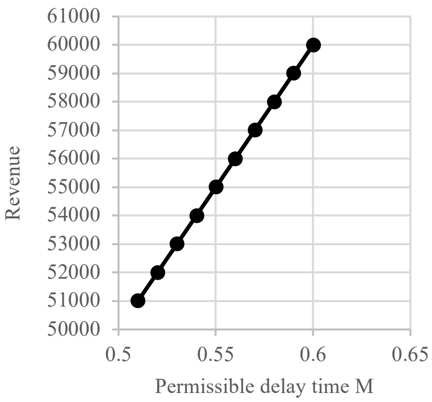


Table & Graph 2: effect of *a* on total revenue.

Let we take the parameters T = 1, M= 0.55, ∝ = 0.2, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	*a*	R_1
0.8	0.11	55001.4
0.8	0.12	55001.53
0.8	0.13	55001.65
0.8	0.14	55001.78
0.8	0.15	55001.91
0.8	0.16	55002.03
0.8	0.17	55002.16
0.8	0.18	55002.29
0.8	0.19	55002.42
0.8	0.2	55002.54

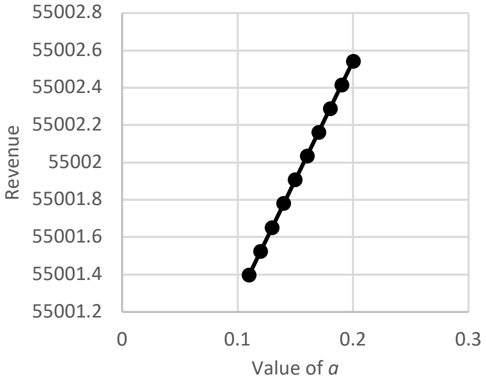
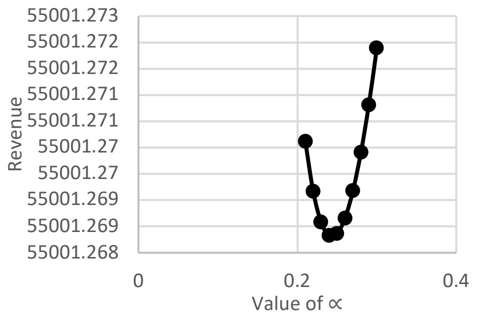


Table & Graph 3: effect of ∝ on total revenue.

Let we take the parameters T = 1, M= 0.55, a = 0.1, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	∝	R_1
0.8	0.21	55001.27
0.8	0.22	55001.27
0.8	0.23	55001.27
0.8	0.24	55001.27
0.8	0.25	55001.27
0.8	0.26	55001.27
0.8	0.27	55001.27
0.8	0.28	55001.27
0.8	0.29	55001.27
0.8	0.3	55001.27



5.2 Case 2: $t_d \leq M \leq T$

Table & Graph 1: effect of M on total revenue.

Let we take the parameters T = 1, a = 0.1, ∝ = 0.2, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	M	R_2
0.6	0.81	81002.47
0.6	0.82	82002.43
0.6	0.83	83002.4
0.6	0.84	84002.37
0.6	0.85	85002.34
0.6	0.86	86002.31
0.6	0.87	87002.28
0.6	0.88	88002.25
0.6	0.89	89002.22
0.6	0.9	90002.19

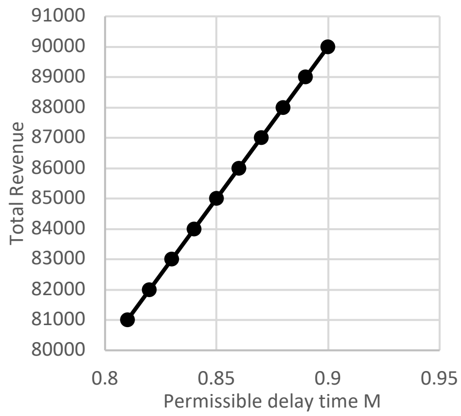


Table & Graph 2: effect of a on total revenue.

Let we take the parameters T = 1, M= 0.8, ∝ = 0.2, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	a	R_1
0.6	0.11	80002.75
0.6	0.12	80003
0.6	0.13	80003.24
0.6	0.14	80003.49
0.6	0.15	80003.74
0.6	0.16	80003.98
0.6	0.17	80004.23
0.6	0.18	80004.48
0.6	0.19	80004.72
0.6	0.2	80004.97

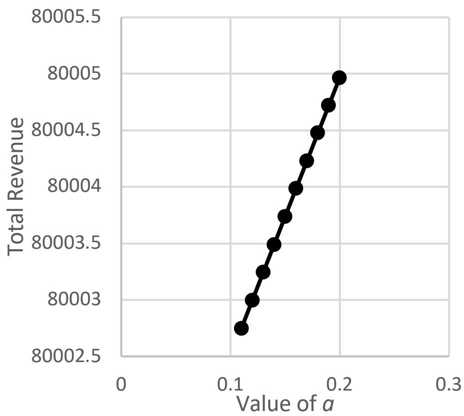
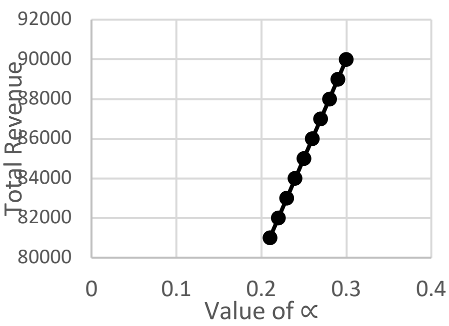


Table & Graph 3: effect of ∝ on total revenue.

Let we take the parameters T = 1, M= 0.8, a = 0.1, Q = 1000, q = 500, Cp = 100, h = 10, p = 20, Ie = 1, Ip = 0.045

t_d	∝	R_1
0.6	0.21	81002.37
0.6	0.22	82002.25
0.6	0.23	83002.14
0.6	0.24	84002.05
0.6	0.25	85001.95
0.6	0.26	86001.87
0.6	0.27	87001.79
0.6	0.28	88001.72
0.6	0.29	89001.65
0.6	0.3	90001.58



6. CONCLUSION AND REMARKS

Time-dependent demand and time-dependent deterioration have been anticipated in the current problem The cost of holding the items is constant and the supplier provides a particular timeframe to make payment.

As per above numerical examples, in case of $0 \leq M \leq t_d$, the increment in total cost with increasing values of M, a, and the parablolic effect of $\propto$ on total cost. In case of $t_d \leq M \leq T$, the total cost increases with increasing values of M, a, and $\propto$.

Deterioration is a natural process and this plays an very effective role in profit and loss in business. The food items, drugs, photographic films, pharmaceuticals, chemicals, and electronic component etc. are very common items, in which deterioration occurs and escalates dramatically over time.

However, it is important to note that the freshness and quality of an item diminish over time after it is produced or purchased. This period of deterioration is referred to as the item's lifetime. It is worth mentioning that the lifetime of each item varies. Unfortunately, most inventory models do not take into account the lifespan of items. This paper addresses this realistic condition by considering the varying demand and studying The consequence of several parameters on the overall revenue.

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