

# Validation of Transformation Laws for Circular and Elliptical Patch Microstrip Antenna using Equivalence Design Concept

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**Abstract:** In past few years, microstrip antennas are highly in demand due to their utilization high frequency and high speed data communication applications. Microstrip antennas are efficient and can easily be accommodated in the device package. Due to their light weight, low profile, low cost, ease to analyze, easy fabrication and compatibility with the integrated circuits, they are found to be the best form of antennas. They are utilized in rectangular, triangular and circular shapes. Transformation of designs laws for feed line of rectangular and triangular patch antennas has already been validated by the authors. This work aims to validate the transformation of Design formulae for Circular and Elliptical patch antennas. Transformation of design laws are based on the 'Equivalence Design Concept' which says that the two designs are said to be equivalent if they result in the same resonance frequency. The simulation results for the circular and elliptical patch antennas are remarkably similar when calculated by the same transformation laws.

**Keywords:** Microstrip antenna, circular patch, elliptical patch, transformation of design, equivalence of design concept

## 1. INTRODUCTION

Conventional formulas are still in practice to design antenna by designers but they are very complex and tedious [1] [2]. The crucial thing while designing antennas of the same frequency using different dielectric materials or dielectric thickness is that the designers are bound to follow the same pattern and have to perform the same mathematical calculations from the very starting to get the same attributes of the antenna what they already have for the another antenna. In this situation, equivalence of design concept is boon for antenna designers. Transformation of design laws for feed line of rectangular and triangular patch antennas has already been validated by the authors [3] [4]. The laws are very easy to remember and carry out very easy mathematical calculations. In this paper, transformation laws are validated for both circular and elliptical shape antennae.

## 1. Validation for circular patch antenna –

The circular patch is designed by using conventional formula [5] and simulated for a frequency 2.4 Ghz.

$$(f_r)_{110} = \frac{1.8412 v_0}{2\pi a_e \sqrt{\epsilon_r}} \quad (1)$$

$$a_p = \frac{F}{\left\{1 + \frac{2h}{\pi \epsilon_r F} \left[ \ln \left( \frac{\pi F}{2h} \right) + 1.7726 \right] \right\}^{\frac{1}{2}}} \quad (2)$$

$$\text{Where } F = \frac{8.791 \times 10^9}{f_r \sqrt{\epsilon_r}} \quad (3)$$

where  $a_e$  and  $a_p$  are effective and physical radius of the circular patch respectively.

$v_0$  = velocity of light in free space

$f_r$  = desired resonant frequency

$h$  = thickness of the substrate

$\epsilon_r$  = dielectric constant of the substrate

The transformation laws for circular patch are given in the next section. Designing and simulation of four sets of antenna having different substrate material with different thicknesses have been done to validate the transformation laws on HFSS simulator software. The simulated results are given under the section simulation results.

## 2. DESIGN

A circular patch antenna design resulting good return loss is considered as the basic design. The dimensions of circular patch are calculated using equation (1) to (3). FR4 Epoxy is chosen as dielectric material having  $\epsilon_r = 4.4$  and dielectric thickness  $h = 1.6$  mm. For this design, the ground plane dimensions would be given as [6] –

$$L_g = W_g = 6h + 2a$$

where,  $L_g$  = length of ground plane  
 $W_g$  = width of ground plane  
 $a$  = radius of circular patch

For all the antenna designs, dimensions (Length and Width) of the substrate and ground plane are kept equal for all the antennas. Inset feeding is provided in this design. The structure is designed for the resonant frequency of 2.4 GHz. Figure 1 shows the structural representation of the circular patch on HFSS simulator.

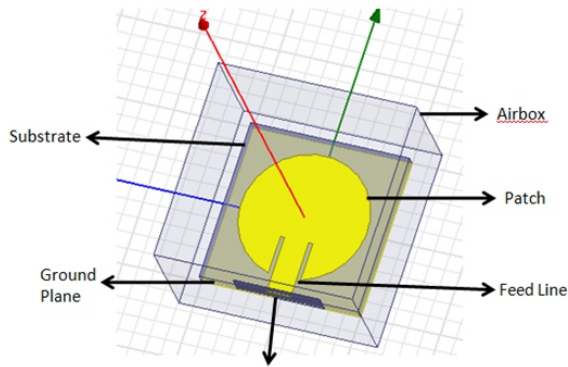


Figure 1. Structural representation of circular shape patch antenna

Now the basic design (Design 1) has to be transformed into another design (Design 2) having different substrate material for the equivalent resonant frequency 2.4 GHz. The transformation laws proposed for the circular patch antenna using equivalence of designs concept are as follows –

- For microstrip antenna, two designs are said to be equivalent if they resonate at the same frequency.
- Assuming that –
- ‘H’ is the main parameter of microstrip antenna
- If ‘H’ is constant between two designs then the designs *may be* equivalent.

Parameters for Design 1, such as dielectric constant of substrate material ( $\epsilon$ ), substrate thickness ( $h$ ) resonant frequency ( $f$ ) are known. To transform Design 1 to another design, following formulas are used keeping resonant frequency equal.

$$a_{ei} = a_{e1} * \psi_i \tag{4}$$

$$a_{pi} = a_{p1} * \psi_i \tag{5}$$

$$H = \frac{1}{c} * f_r * h * \sqrt{\epsilon_r}$$

$$h_i = h_1 * \psi_i \tag{6}$$

Where, 
$$\psi_i = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{ri}}}$$

$h_i$  = Thickness of the substrate used for  $i^{th}$  Design.  
 $h_1$  = Thickness of the substrate used for Design 1.  
 $a_{p1}$  = Physical radius of Design 1.  
 $a_{pi}$  = Physical radius of  $i^{th}$  Design.  
 $a_{e1}$  = Effective radius of Design 1.  
 $a_{ei}$  = Effective radius of  $i^{th}$  Design.  
 $\epsilon_{r1}$  and  $\epsilon_{ri}$  are the dielectric constants of design 1 and  $i^{th}$  design respectively.  
 where  $i=2,3,4 \dots$  and so on.

### 3. METHODOLOGY

Using equation (4) to (6) parameters of the new design having different substrate material are calculated. Then all the antenna structures are designed on HFSS simulator. The basic design is then transformed into three new designs having different substrate materials and the results are summarized in table 1 which is shown as below.

TABLE 1. Return Loss table for the antenna designs using Transformation Laws

Substrate Material	( $\epsilon_r$ )	h (mm)	$f_r$ (GHz)	$S_{11}$ (in dB)
Fr4 Epoxy	4.4	1.6	2.45	-44.0598
Rogers RT/Duriod 5880 (tm)	2.2	2.26 1.37	2.40	-32.2470
Rogers TMM 6 (tm)	6		2.43	-42.7694
Arlon AD1000(tm)	10.2	1.05	2.45	-36.1297

The simulation results shown in the above table shows that all the transformed structures are resonating on approximately the same designed frequency of 2.4 Ghz.

### 4. SIMULATION RESULTS

The basic design (Design 1) is designed using FR4 epoxy as a substrate material having dielectric thickness of 1.6 mm at resonant frequency 2.4 GHz. The plot for return loss versus frequency for Design 1 using HFSS is shown in figure 2.

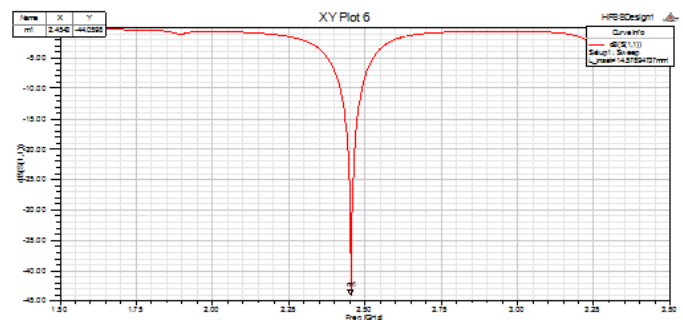


Figure 2. Return loss versus frequency plot for circular patch antenna for Design 1

Now Design 1 is transformed into Design 2. Rogers/Duriod 5880(tm) is chosen as dielectric material having  $\epsilon_r = 2.2$ . Parameters of Design 2 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 2 is shown in figure 3.

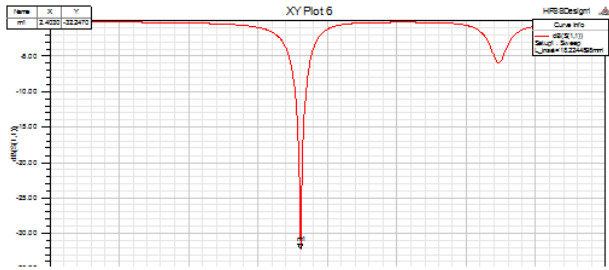


Figure 3. Return loss versus frequency plot for circular patch antenna for Design 2

Similarly Design 1 is transformed into Design 3. Rogers TMM 6 (tm) is chosen as dielectric material having  $\epsilon_r = 6$ . Parameters of Design 3 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 3 is shown in figure 4.

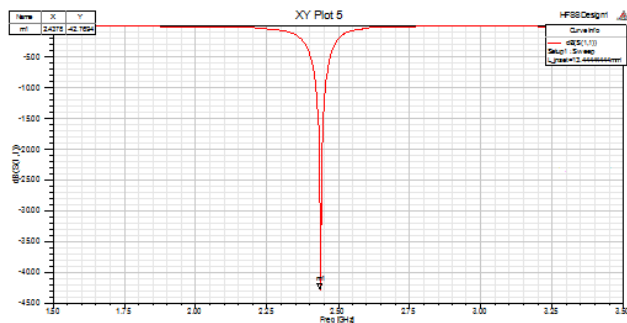


Figure 4. Return loss versus frequency plot for circular patch antenna for Design 3

Again, Design 1 is transformed into Design 4. Arlon AD1000(tm) is chosen as dielectric material having  $\epsilon_r = 10.2$ . Parameters of Design 4 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 4 is shown in figure 5.

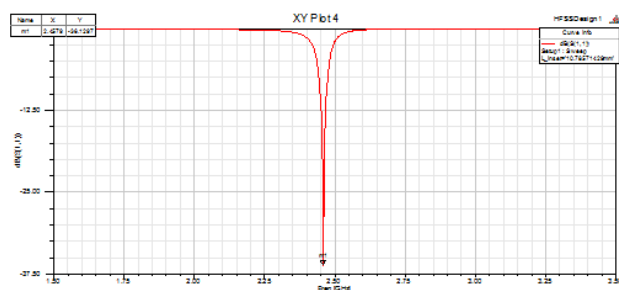


Figure 5. Return loss versus frequency plot for circular patch antenna for Design 4

**Validation for elliptical patch antenna –**

The elliptical patch is designed by using conventional formula [8] and simulated for a frequency 1 GHz. The empirical formulas for calculation of dual resonance frequency using approximated Mathieu function are listed below [7-9]–

$$a_{eff} = a \left[ 1 + \frac{2h}{\pi \epsilon_r a} \left\{ \ln \left( \frac{a}{2h} \right) + (1.41 \epsilon_r + 1.77) + \frac{h}{a} (0.268 \epsilon_r + 1.65) \right\} \right]^{\frac{1}{2}} \quad (7)$$

$$f_{11}^{e,0} = \frac{15}{\pi e a_{eff}} \sqrt{\frac{q_{11}^{e,0}}{\epsilon_r}} \quad (8)$$

$$q_{11}^e = -0.0049 e + 3.7888 e^2 - 0.7278 e^3 + 2.314 e^4 \quad (9)$$

$$q_{11}^0 = -0.0063 e + 3.8316 e^2 - 1.1351 e^3 + 5.2229 e^4 \quad (10)$$

- where a = semi major axis
- h = height of dielectric substrate (in cms)
- $\epsilon_r$  = permittivity of dielectric substrate
- $a_{eff}$  = effective semi-major axis (in cms)
- e = eccentricity of elliptical patch
- $q_{11}^{e,0}$  = Approximated Mathieu function of the dominant ( $TM_{11}^{e,0}$ ) mode

$f_{11}^{e,0}$  = Dual resonance frequency  
 For an ellipse of semi major axis ‘a’ and semi minor axis ‘b’, the foci are at  $\pm c$ , where

$$c = \sqrt{a^2 - b^2} \text{ or } c = a \sqrt{1 - \left( \frac{b}{a} \right)^2} \quad (11)$$

and 
$$a = \frac{p}{f \sqrt{\mu \epsilon}} \quad (12)$$

where  $\mu = \mu_0 \mu_r$  and  $\epsilon = \epsilon_0 \epsilon_r$  are the permeability and permittivity of the substrate respectively.  $p = 0.275$ . The eccentricity of the ellipse is defined as  $e = c/a$

Another formula to calculate the effective major radius of elliptical patch is chosen equal to half of the effective wavelength which is given by [10];

$$a_e = \frac{c}{4f_r \sqrt{\epsilon_r e}} \quad (13)$$

**5. DESIGN**

An elliptical patch antenna design resulting good return loss is considered as the basic design. The dimensions of elliptical patch are calculated using equation (13). FR4 Epoxy is chosen as dielectric material having  $\epsilon_r = 4.4$  and dielectric thickness  $h = 1.6$  mm. For this design, the ground plane dimensions would be given as [6]–

$$\begin{aligned} Lg &= 6h + 2a \\ Wg &= 6h + 2b \end{aligned}$$

Where,  $L_g$  = length of ground plane  
 $W_g$  = width of ground plane  
 $a$  = semi-major axis  
 $b$  = semi-minor axis

For all the antenna designs, dimensions (Length and Width) of the substrate and ground plane are kept equal for all the antennas. The aspect ratio of elliptical patch is taken 1.3. An *inset feed* type feed is used in all antenna designs and feeding is provided perpendicular to minor axis. The structure is designed for the resonant frequency of 1 GHz. Figure 6 shows the structural representation of the elliptical patch on HFSS simulator.

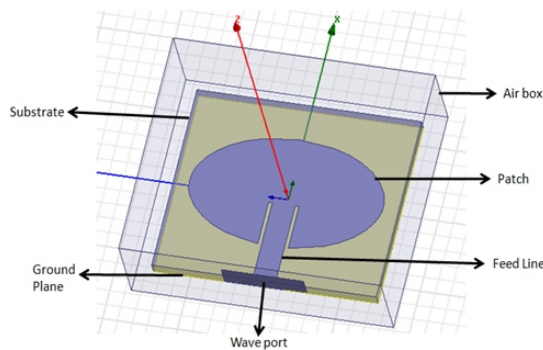


Figure 6. Structural representation of elliptical shape patch antenna

Now the basic design (Design 1) has to be transformed into another design (Design 2) having different substrate material for the equivalent resonant frequency 1 GHz. The transformation laws for elliptical patch are same as for circular patch which are given in equation (4), (5) and (6).

**6. METHODOLOGY**

Using equation (13), dimensions of the new design having different substrate material are calculated. Then all the antenna structures are designed on HFSS simulator. The basic design is then transformed into three new designs having different substrate materials and the results are summarized in table 2 which is shown as below.

**TABLE 2.** Return Loss table for the antenna designs using Transformation Laws

Substrate Material	( $\epsilon_r$ )	h (mm)	$f_r$ (GHz)	$S_{11}$ (in dB)
Fr4 Epoxy	4.4	1.6	0.9994	-52.7687
Rogers RT/Duriod 5880	2.2	2.26	0.9981	-35.8741
Rogers TMM 6	6	1.37	0.9924	-61.0544
Arlon AD1000	10.2	1.05	0.9957	-45.9052

The simulation results shown in the above table shows that all the transformed structures are resonating on approximately the same designed frequency of 1 GHz.

**7. SIMULATION RESULTS**

The basic design (Design 1) is designed using FR4 epoxy as a substrate material having dielectric thickness of 1.6 mm at resonant frequency 1 GHz. The plot for return loss versus frequency for Design 1 using HFSS is shown in figure 7.

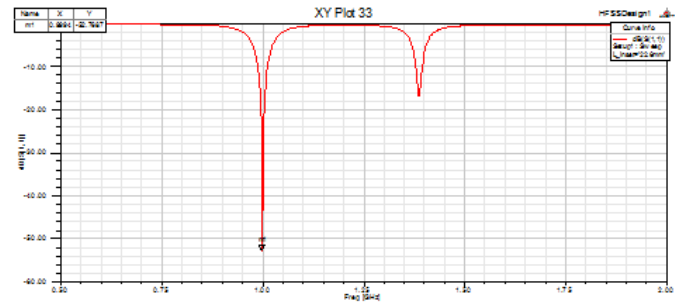


Figure 7. Return loss versus frequency plot for elliptical patch antenna for Design 1

Now Design 1 is transformed into Design 2. Rogers/Duriod 5880(tm) is chosen as dielectric material having  $\epsilon_r = 2.2$ . Parameters of Design 2 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 2 is shown in figure 8.

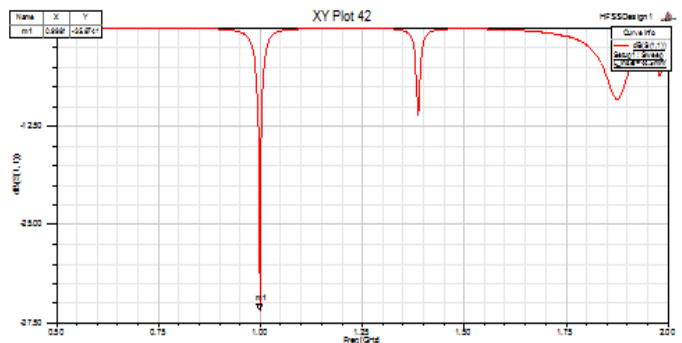


Figure 8. Return loss versus frequency plot for elliptical patch antenna for Design 2

Similarly Design 1 is transformed into Design 3. Rogers TMM 6 (tm) is chosen as dielectric material having  $\epsilon_r = 6$ . Parameters of Design 3 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 3 is shown in figure 9.

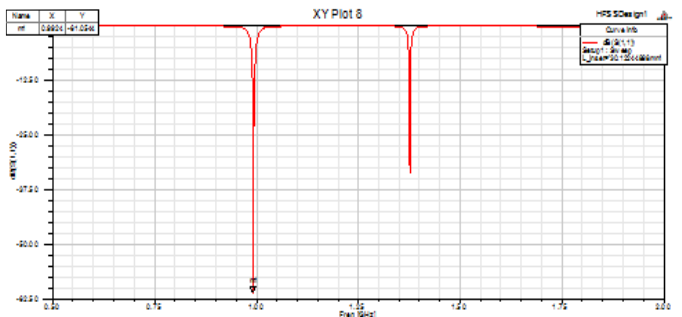


Figure 9. Return loss versus frequency plot for elliptical patch antenna for Design 3

Again, Design 1 is transformed into Design 4. Arlon AD1000(tm) is chosen as dielectric material having  $\epsilon_r = 10.2$ . Parameters of Design 4 are calculated using transformation laws shown in equation (4) to (6). The plot for return loss versus frequency for Design 4 is shown in figure 10.

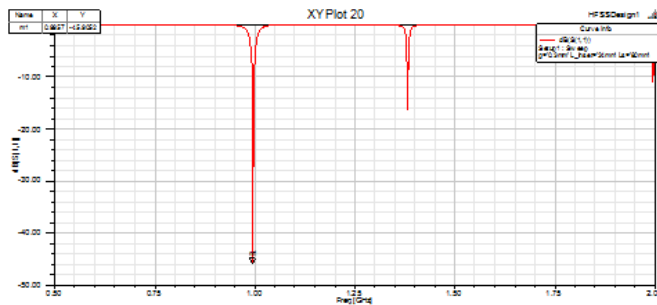


Figure 10. Return loss versus frequency plot for elliptical patch antenna for Design 4

## 8. CONCLUSION

The transformation of design is an excellent idea as the proposed laws are simple to utilize. Scaling factor is a vital parameter. It gives the vital connection between any two designs. It prompts another method for taking a gander at the designing field of microstrip antennas. Individual values of  $\epsilon_{r1}$  and  $\epsilon_{r2}$  do not make any difference. The critical thing is the square root of their proportion. In this manner, increment or decrement in the values  $\epsilon_{r1}$  can be counterbalanced by increasing or decreasing the value of  $\epsilon_{r2}$  with the end goal that the scaling factor remains steady. This gives another level of opportunity in microstrip antenna designing. According to the new formula, 'H' is the essential parameter of MSA. By utilizing these formulas, it becomes very easy to transform one antenna design to another. They are very simple and less complex. By incorporating these formulas, one

needs not to make the entire calculations again to find out the dimensions of the new antennas as it provides a quick relation between the two designs.

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