Data Driven ANN-ML based Levenberg Marquardt Algorithm for Fractional Coupled Differential Equations

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Abstract- In this paper we have discussed the Artificial neural network based Levenberg Marquardt algorithm (ANN-LMA) for handled coupled differential equations. The system of equation is further solved by Atangana-Toufik scheme. The numerical results are trained by using the back propogation method. Comparison of the both methods are shown through graphs.

Keywords– Coupled system, ANN, Atangana-Toufik method, Training.

1. INTRODUCTION

An emerging field of technology called "smart computing" incorporates intelligence into computer systems to increase their autonomy, efficiency, and adaptability. It makes use of cutting-edge technologies including blockchain, cloud computing, edge computing, big data analytics, artificial intelligence (AI), machine learning (ML), and the Internet of Things (IoT). These systems can adapt to changing conditions, learn from data, and make decisions with little assistance from humans. Smart computing systems are intended to be self-learning, context-aware, and sensitive to real-time input, in contrast to traditional computing, which is static and rule-based [1-3]. Large-scale data processing and analysis from linked devices, sensors, and users is the foundation of smart computing. IoT allows physical objects to share data and communicate, while AI and ML allow the systems to identify trends and make predictions. With low latency and scalability guaranteed, cloud and edge computing offer the infrastructure required for effective data processing and storage. Data exchanges in smart environments are made more secure and transparent by blockchain technology. In the future, smart computing is expected to evolve further with advancements like neuromorphic computing, quantum computing, digital twins, and sustainable (green) computing. These innovations will lead to even more powerful and efficient systems capable of solving complex real-world problems in a smarter and more human-like way.

The ANN's backgrounds may be traced back to the 1940s, when early models such as the McCulloch-Pitts neuron were developed. The perceptron was introduced in the 1950s, and backpropagation gained popularity in the 1980s. However, delayed convergence and problems with local minima were common problems with old training techniques. In order to solve this, the Levenberg–Marquardt (LM) algorithm was modified for ANN training. It was first created in the 1940s and 1960s to solve nonlinear least squares issues. Particularly for small to medium-sized networks, LM offers faster and more stable convergence by acting as a hybrid between the gradient descent and Gauss-Newton approaches. Combining the modeling prowess of artificial neural networks with the optimization effectiveness of the Levenberg–Marquardt algorithm, Levenberg Marquardt is a potent computational method. Inspired by the neural architecture of the human brain, artificial neural networks (ANNs) can learn intricate nonlinear correlations from data [4-5].

In this paper we have discussed the following coupled system of equation with Caputo derivative

$$^{\alpha}_{t}D y_1(t) = -a_1 y_1(t) + b_1 sin(y_2((t - \delta)) \quad (1)$$
$$^{\alpha}_{t}D y_2(t) = -c_1 y_1{}^2(t) + d_1 y_2(t) + y_3(t) \quad (2)$$
$$^{\alpha}_{t}D y_3(t) = -e_1 y_1(t) y_3(t) + f_1 y_2(t) \quad (3)$$
$$0 \leq \alpha < 1$$

Where $a_1, b_1, c_1, d_1, e_1,$ and f_1 are constants and δ be the time delays parameter.

2. ATANGANA TOUFIK METHOD (ATM)

It is a numerical method based on two steps lagranges multipliers polynomials. This method is free from discretization and assumpstions and delivers accuracy, stability, efficiency, and convergence to solve fractional differential equations of constant and variable orders [6-10].

3. ANN-LM METHOD

The artificial neural network based Levenberg-Marquardt algorithm is a controlling optimization technique to trained the neural network and minimizing the error providing input and output datas by least sqaure method. The ANN-LMA is hybrid technique combined from Gradient Descent and Gauss-Newton method [11-17].

In this paper we have derived 500 data points from the atangana-toufik method and trained the data by ANN-

LMA. Out of which 70% is used for training, 15% is for testing and 15% is aimed at validating. We inspect the mean square error (MSE), Comaparative study between the Numerical methods and ANN-LM, Error plots and regression analysis. Through MATLAB nntool/nftool command we trained the obtained data and compared with the numerical method.

Figure 1: Comparative study of y_1

Figure 2: Comparative study of y

Figure 3: Comparative study of y_3

Theses are the step-by-step method for trained the data:

Step-1 Create the datas

Step-2 Use Matlab builtin function nftool/nntool to store input (independent variable(s)) and ouput (dependent variable(s)) the data.

Step-3 Use Activation functions (Sigmoid, Tanh, linear etc.) to approximate the function datas.

Step-4 Create ANN-LM structure.

Step-5 Established Training Parameters.

Step-6 Train the network (Input/Output).

Step-7 Examine and Predict the output data.

Step-8 Evaluate the performance of ANN-LM.

Step-9 See the results obtained by ANN and Numerical method.

Step-10 Assess the Comapritive results, MSE, Validating performance, Error histogram and Regression analysis.

4. CONCLUSIONS

The system of equations is solved by the Atangana-Toufik scheme and the results are trained by using the ANN-LMA.

We have obtained the 500 data points by ATM and trained the input and output by machine learning based backporpogation algorithm. The parametrs are assumes as $a_1 = 0.1$, $b_1 = 0.2$, $c_1 = 0.01$, $d_1 = 0.1, e_1 = 0.1$, and $f_1 = 0.2$.

Comparative study between the Atangana-toufik meyod and ANN-LMA are shown in figures 1-3. Figure 4 plotted for MSE of y_1 shows a best validating performance 1.0477e-10 at 1169 epochs. Figure 5 describe the values of Gradient (first derivatives/Jacobian wrto weight function) and Mu (restraining aspect of governing Gradient Descent and Gauss-Newton method) as (1.4423e-07, 1e-09) respectively. Figure 6 portrays error histogram between the errors and validating, testing, and training datas. Figure 7 is plotted for regression analysis where $\bar{y}$ represent the average of the true values, the range of R^2 is between 0 to 1. $R \approx 1$ designates an outstanding agreement between the predicted values and exact values. Figure 8 designed for MSE of y_2 shows a best validating performance 1.2635e-10 at 900 epochs. Moreover Figure 9 label the values of Gradient and Mu as (1.6442e-07, 1e-09) respectively. Figure 10 depicts error histogram between the errors and validating, testing, and training datas. Figure 11 is plotted for regression analysis between the indepent variable and dependent variable and $R \approx 1$ emphasise an excellent realtion with the forecast values and exact values. The schematic diagram is shown in Figure 12.
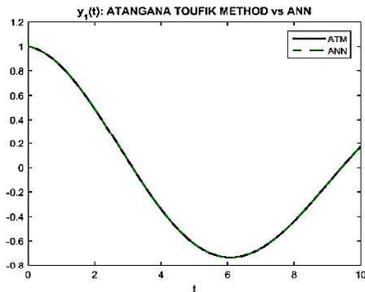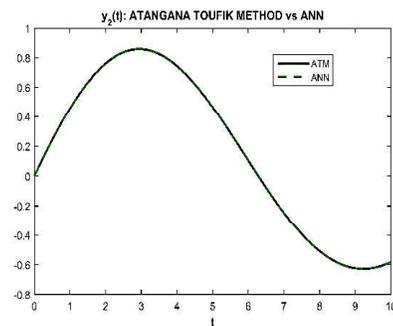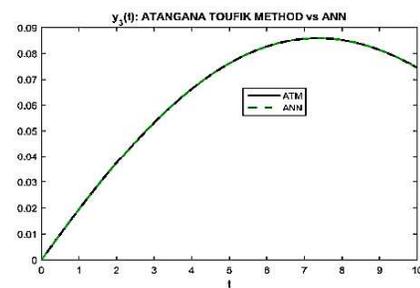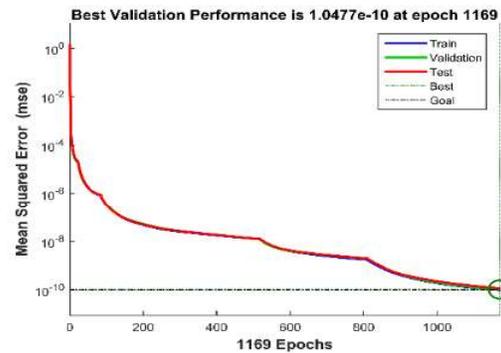

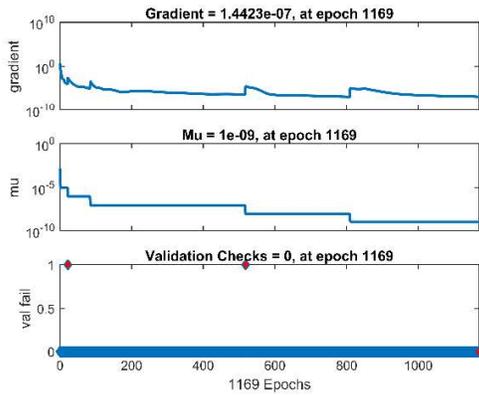**Figure 4:** Validating the result y_1 by ANN-LMA

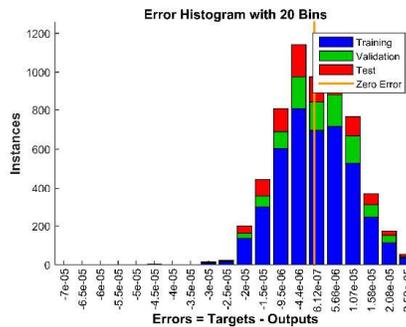**Figure 5:** Evaluation of gradient, mu of y_1 by ANN-LMA

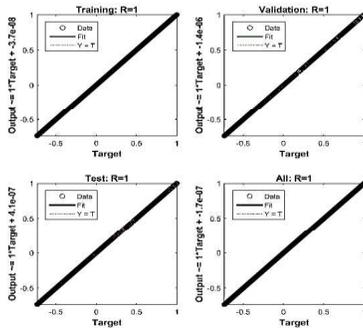**Figure 6:** Error histogram of y_1 by ANN-LMA
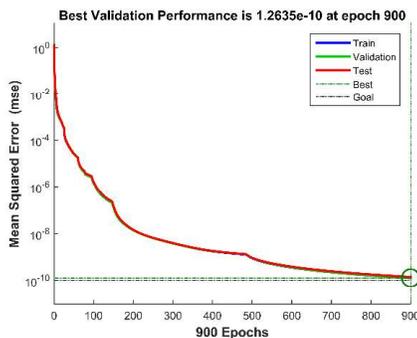

Figure 7: Regression analysis of y_1 by ANN-LMA

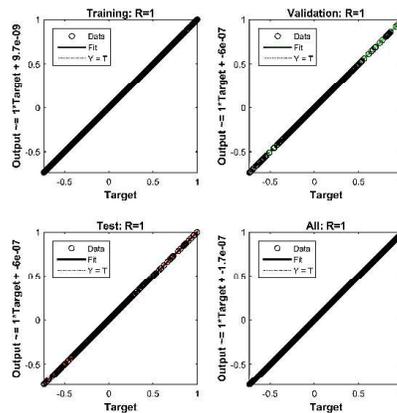**Figure .8:** Valiadating the result y_2 by ANN-LMA

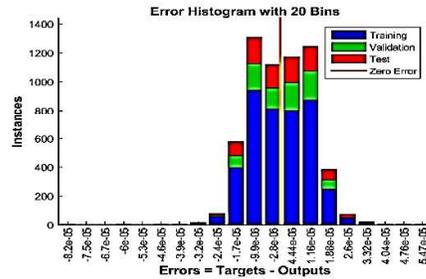**Figure 9:** Evaluation of gradient, mu of y_2 by ANN-LMA

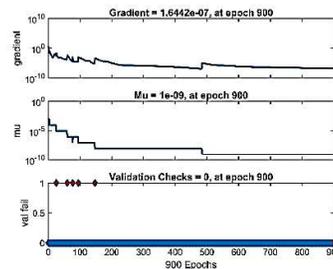**Figure 10:** Error histogram of y_2 by ANN-LMA

Figure 11: Regression analysis of y_2 by ANN-LMA
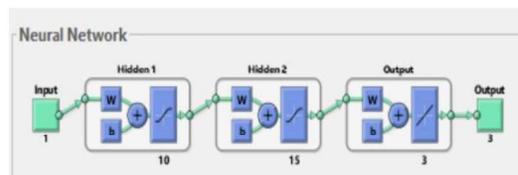

Figure 12: Neural netwok diagram

REFERENCES

[1]. P. Gund, P. Mane, H. Padalikar, S. Thorat, S. Patil, and K. Dharmadhikari, "Exploring students' perspectives: Leveraging smart computing for online and offline education optimization," in Proceedings of the 2024 OPJU International Technology Conference (OTCON) on Smart Computing for Innovation and Advancement in Industry 4.0, (2024), pp. 1–6.

[2]. M. P. Krishna, S. Mehta, S. Verma, and S. Rane, "Mixed reality in smart computing education system," in Proceedings of the

2018 International Conference on Smart Systems and Inventive Technology (ICSSIT), (2018), pp. 72–75.

[3]. A. Ghosh, D. Chakraborty, and A. Law, "Artificial intelligence in Internet of Things," CAAI Transactions on Intelligence Technology, (2018), vol. 3, no. 4, pp. 208–218.

[4]. R. T. Yarlagadda, "Internet of Things and artificial intelligence in modern society," International Journal of Creative Research Thoughts (IJCRT), (2018), ISSN: 2320-2882.

[5]. V. E. Balas, R. Kumar, and R. Srivastava (Eds.), Recent Trends and Advances in Artificial Intelligence and Internet of Things, (2020), Cham, Switzerland: Springer International Publishing.

[6]. G. Lazaroiu, A. Androniceanu, I. Grecu, G. Grecu, and O. Negurită, "Artificial intelligence-based decision-making algorithms, Internet of Things sensing networks, and sustainable cyber-physical management systems in big data-driven cognitive manufacturing," Oeconomia Copernicana, (2022), vol. 13, no. 4, pp. 1047–1080.

[7]. A. Bhargava, D. L. Suthar, and D. Jain, "A new solution approach to proportion delayed and heat-like fractional partial differential equations," Partial Differential Equations in Applied Mathematics, (2024), vol. 12, p. 100948.

[8]. J. Singh, "Analysis of fractional blood alcohol model with composite fractional derivative," Chaos, Solitons & Fractals, (2020), vol. 140, p. 110127.

[9]. D. Jain and A. Bhargava, "On certain fractional kinetic equations involving Laguerre polynomials," Advances in Mathematics: Scientific Journal, (2020), vol. 9, no. 9, pp. 7075–7084.

[10]. D. Jain, A. Bhargava, and S. Gupta, "A new approach to population growth model involving a logistic differential equation of fractional order," Critical Reviews™ in Biomedical Engineering, (2025), vol. 53.

[11]. E. Specht, T. Redemann, and N. Lorenz, "Simplified mathematical model for calculating global warming through anthropogenic CO_2," International Journal of Thermal Sciences, (2016), vol. 102, pp. 1–8.

[12]. A. Aziz, S. A. H. Shah, H. M. S. Bahaidarah, T. Zamir, and T. Aziz, "Advanced neural network modeling with Levenberg–Marquardt algorithm for optimizing tri-hybrid nanofluid dynamics in solar HVAC systems," Case Studies in Thermal Engineering, (2025), vol. 65, p. 105609.

[13]. F. A. Shah, T. Zamir, N. S. Akbar, and A. Mikhaylov, "Levenberg–Marquardt design for analysis of Maxwell fluid flow on ternary hybrid nanoparticles passing over a Riga plate under convective boundary conditions," Results in Engineering, (2024), vol. 24, p. 103502.

[14]. J. M. H. Barakat, Z. Al Barakeh, and R. Ghandour, "Levenberg–Marquardt analysis of MHD hybrid convection in non-Newtonian fluids over an inclined container," Eng, (2025), vol. 6, no. 5, p. 92.

[15]. Mahariq, K. Ullah, M. Fiza, A. U. Jan, H. Ullah, S. Islam, and S. M. Al-Mekhlafi, "Levenberg–Marquardt recurrent neural network for heat transfer in ternary hybrid nanofluid flow with nonlinear heat source–sink," Advances in Mechanical Engineering, (2025), vol. 17, no. 6, p. 16878132251341968.

[16]. Faisal and A. Rauf, "Analysis of the tri-hybrid Maxwell nanofluid flow with temperature-dependent thermophysical characteristics on static and dynamic surfaces employing Levenberg–Marquardt artificial neural networks," Numerical Heat Transfer, Part B: Fundamentals, (2024), pp. 1–27.

[17]. F. Ali, M. Awais, A. Ali, N. Vrinceanu, Z. Shah, and V. Tirth, "Intelligent computing with Levenberg–Marquardt artificial neural network for carbon nanotubes–water flow between stretchable rotating disks," Scientific Reports, (2023), vol. 13, no. 1, p. 3901.