Intelligent Computational Machine Learning Approach for solving 3D Heat-like Equation

Deepika Khandelwal[1], Sumit Gupta[2]
[1,2] Department of Mathematics, Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur, India
Email: **deepika.khandelwal@skit.ac.in, sumit.gupta@skit.ac.in**
Received 01.08.2025 **received in revised form** 15.10.2025, **accepted** 29.10.2025
DOI: 10.47904/IJSKIT.15.2.2025.84-87

Abstract- This manuscript deal with the three-dimensional (3D) heat like equations solved by the Artificial neural network based Levenberg Marquardt algorithm (ANN-LMA) The numerical result is handleled by the Taangana Toufik scheme. The comparison between the ANN and semi-analytical trechniques are shown in tables.
Keywords– Coupled system, ANN, Atangana-Toufik method, Training.

1. INTRODUCTION

The Machine learning based Artificial neural network (ANN) is a type of computational prototypical inspired by the functioning of biological neural network in human brain. It is a type of deep learning that uses to acquire about the realtionships of input/output datas and understanding the complex patterns. The principal components of ANN are Neurons (Nodes), weights, Activation functions, Back Propogation, Loss calculations, Optimzation and Training of the input datas. Nowadays, ANN are widely used in Fractal-Fractional type differential equation, Nanofluid dynamics, Mathematical modelling etc., [1-8].

2. ATANGANA TOUFIK METHOD (ATM)

It is a numerical method based on two steps lagranges multipliers polynomials. This method is free from discretization and assumpstions and delivers accuracy, stability, efficiency, and convergence to solve fractional differential equations of constant and variable orders [9-17].

3. ANN-LM METHOD

The artificial neural network based Levenberg-Marquardt algorithm is a controlling optimization technique to trained the neural network and minimizing the error providing input and output datas by least sqaure method. The ANN-LMA is hybrid technique combined from Gradient Descent and Gauss-Newton method.

In this paper we have derived 500 data points from the atangana-toufik method and trained the data by ANN-LMA. Out of which 70% is used for training, 15% is for testing and 15% is aimed at validating. We inspect the mean square error (MSE), Comaparative study between the Numerical methods and ANN-LM, Error plots and regression analysis. Through MATLAB nntool/nftool command we trained the obtained data and compared with the numerical method.

Theses are the step-by-step method for trained the data:

Step-1 Create the datas
Step-2 Use Matlab builtin function nftool/nntool to store input (independent variable(s)) and ouput (dependent variable(s)) the data.
Step-3 Use Activation functions (Sigmoid, Tanh, linear etc.) to approximate the function datas.
Step-4 Create ANN-LM structure.
Step-5 Established Training Parameters.
Step-6 Train the network (Input/Output).
Step-7 Examine and Predict the output data.
Step-8 Evaluate the performance of ANN-LM.
Step-9 See the results obtained by ANN and Numerical method.
Step-10 Assess the Comapritive results, MSE, Validating performance, Error histogram and Regression analysis.

4. THREE-DIMENSIONAL (3D) HEAT-LIKE EQUATIONS

4.1 Consider the following heat-like equation [14,15]

$$u_t(x,y,z,t) = \alpha(u_{xx} + u_{yy} + u_{zz})$$

with the initial conditon

$$u(x,y,z,0) = x^2 - x + y^2 - y + z^2 - z$$

At $\alpha = -1$, the exact solution is given by

$$u(x,y,z,t) = x^2 - x + y^2 - y + z^2 - z + 6t.$$

Table 1: Numerical solutions of example 4.1

u(x,y,z,t)	Exact solution	Ref. [15]	Present solution (ANN)
(0,0,0,0)	0.00000	0.00000	0.00000
(0.1,0.1,0.1, 0.1)	0.33000	0.33000	0.33000
(0.2,0.2,0.2, 0.2)	0.72000	0.72000	0.72000
(0.3,0.3,0.3, 0.3)	1.17000	1.17000	1.17000
(0.4,0.4,0.4, 0.4)	1.68000	1.68000	1.68000
(0.5,0.5,0.5, 0.5)	2.25000	2.25000	2.25000
(0.6,0.6,0.6, 0.6)	2.28000	2.28000	2.28000
(0.7,0.7,0.7, 0.7)	3.57000	3.57000	3.57000

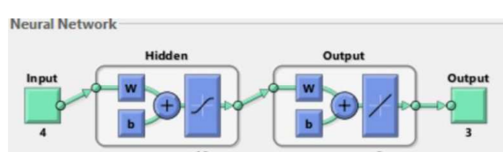

Figure 1: Schematic diagram of ANN

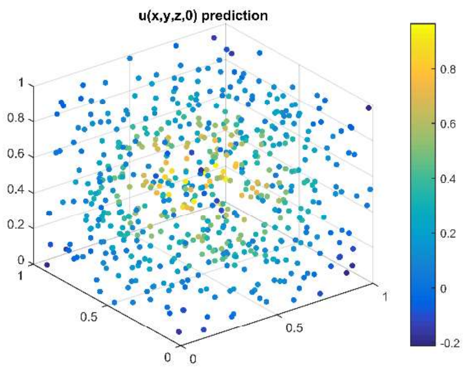

Figure 2: Solution prediction of u through ANN

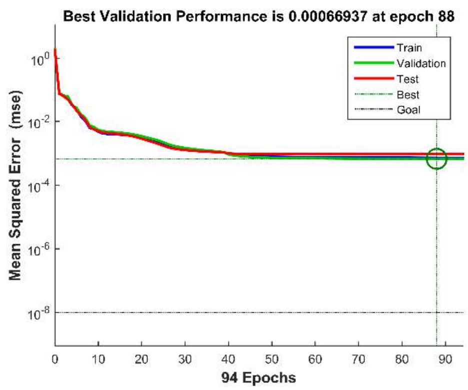

Figure 3: MSE of u through ANN

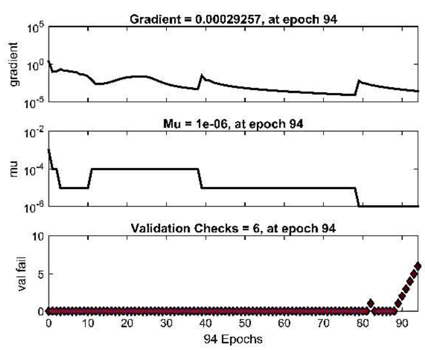

Figure 4: Gradient and Mu of u by ANN

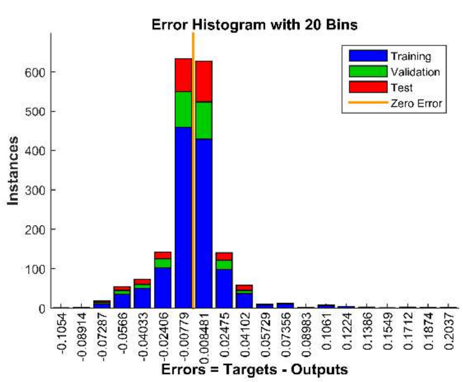

Figure 5: Error histogram of u by ANN

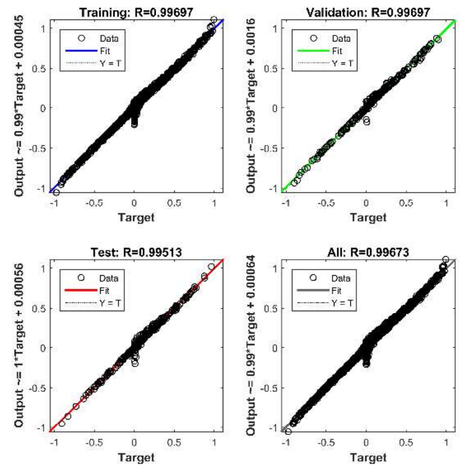

Figure 6: Regression analysis of u by ANN

4.2 Consider the following heat-like equation [*14,15*]

$$u_t(x,y,z,t) = \alpha(u_{xx} + u_{yy} + u_{zz})$$

with the initial conditon

$$u(x,y,z,0) = e^{x+y+z}$$

At $\alpha = -1$, the exact solution is given by $u(x,y,z,t) = e^{x+y+z+3t}$.

Table 2: Numerical solutions of example 4.2

u(x,y,z,t)	Exact solution	Ref.[15]	Present solution (ANN)
(0,0,0,0)	1.00000	1.00000	1.00000
(0.1,0.1,0.1, 0.1)	0.977122	0.977122	0.977122
(0.2,0.2,0.2, 0.2)	0.895094	0.895094	0.895094
(0.3,0.3,0.3, 0.3)	0.728847	0.728847	0.728847
(0.4,0.4,0.4, 0.4)	0.445108	0.445108	0.445108
(0.5,0.5,0.5, 0.5)	0.000000	0.00000	0.0000000
(0.6,0.6,0.6, 0.6)	-0.66402	-0.66402	-0.66402
(0.7,0.7,0.7, 0.7)	-1.62207	-1.62207	-1.62207

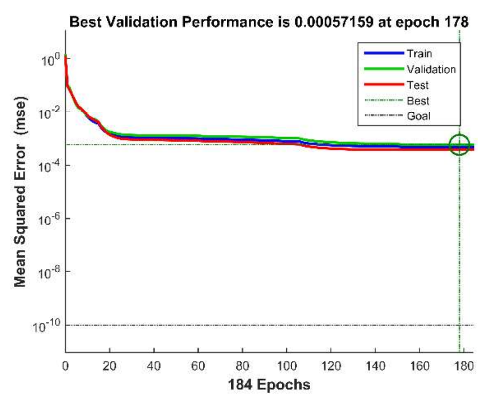

Figure 7: MSE of u through ANN

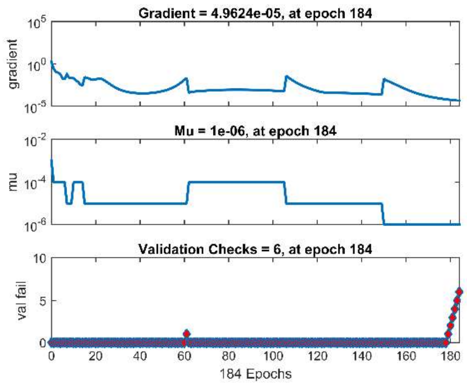

Figure 8: Gradient and Mu of u through ANN

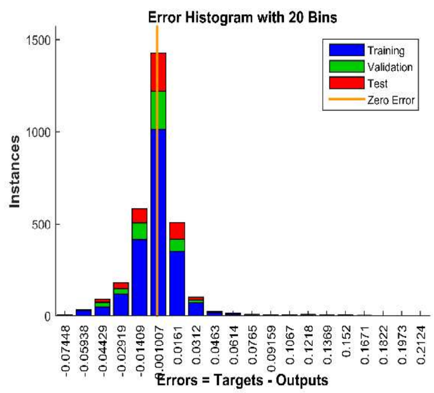

Figure 9 ; Error Histogram of u through ANN

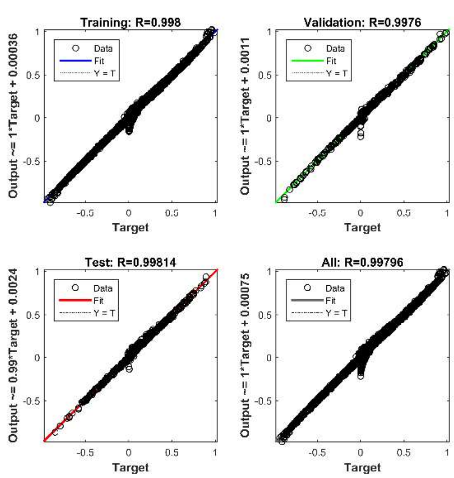

Figure 10: Regression analysis of u through ANN

5. CONCLUSIONS

In this paper we have discussed the three-dimensional heat-like equations with initial conditions. These equations have been solved by the ANN based back propogation algorithm. Table 1 and 2 are shown the accuracy of ANN approach with the semi analytical and xeact values. Schematic diagram of ANN was designed in Figure 1. The region of solution prediction $u(x, y, z, 0)$ is depicted in Figure 2. In Figure 3 the mean square error MSE of u shows a best validating performance 6.637e-10 at 94 epochs. Figure 4 designate the principles of Gradient (first derivatives/Jacobian wrto weight function) and Mu (restraining aspect of governing Gradient Descent and Gauss-Newton method) as (1.235e-05, 1e-07) respectively. Figure 5 depicts error histogram between the errors and validating, testing, and training datas. Figure 6 is plotted for regression analysis where $\bar{y}$ represent the average of the true values, the range of R^2 is between 0 to 1. $R \approx 1$ entitles an excellent arrangement among the predicted values and exact values. Figure 7 calculated for MSE of u (example 4.2) demonstrate a best validating performance 5.7e-05 at 184 epochs. Additionally Figure 8 describes the values of Gradient and Mu as (4.96e-05, 1e-06) correspondingly. Figure 9 represents error histogram between the errors and validating, testing, and training datas of 184 epochs. Figure 10 is plotted for regression analysis between the indepent variable and dependent variable and $R \approx 1$ emphasise an excellent realtion with the predict and exact solutions

REFERENCES

[1]. Aziz, S. A. H. Shah, H. M. S. Bahaidarah, T. Zamir, and T. Aziz, “Advanced neural network modeling with Levenberg–Marquardt algorithm for optimizing tri-hybrid nanofluid dynamics in solar HVAC systems,” Case Studies in Thermal Engineering, (2025), vol. 65, p. 105609.

[2]. F. A. Shah, T. Zamir, N. S. Akbar, and A. Mikhaylov, “Levenberg–Marquardt design for analysis of Maxwell fluid flow on ternary hybrid nanoparticles passing over a Riga plate under convective boundary conditions,” Results in Engineering, (2024), vol. 24, p. 103502.

[3]. J. M. H. Barakat, Z. Al Barakeh, and R. Ghandour, “Levenberg–Marquardt analysis of MHD hybrid convection in non-Newtonian fluids over an inclined container,” Eng, (2025), vol. 6, no. 5, p. 92.

[4]. A. Mahariq, K. Ullah, M. Fiza, A. U. Jan, H. Ullah, S. Islam, and S. M. Al-Mekhlafi, “Levenberg–Marquardt recurrent neural network for heat transfer in ternary hybrid nanofluid flow with nonlinear heat source–sink,” Advances in Mechanical Engineering, (2025), vol. 17, no. 6, p. 16878132251341968.

[5]. Faisal and A. Rauf, “Analysis of the tri-hybrid Maxwell nanofluid flow with temperature-dependent thermophysical characteristics on static and dynamic surfaces employing Levenberg–Marquardt artificial neural networks,” Numerical Heat Transfer, Part B: Fundamentals, (2024), pp. 1–27.

[6]. F. Ali, M. Awais, A. Ali, N. Vrinceanu, Z. Shah, and V. Tirth, “Intelligent computing with Levenberg–Marquardt artificial neural network for carbon nanotubes–water flow between stretchable rotating disks,” Scientific Reports, (2023), vol. 13, no. 1, p. 3901.

[7]. M. Shoaib, T. Rafia, M. A. Z. Raja, W. A. Khan, and M. Waqas, “Further analysis of double-diffusive flow of nanofluid through a porous medium situated on an inclined plane: AI-based Levenberg–Marquardt scheme with backpropagated neural network,” Journal of the Brazilian Society of Mechanical Sciences and Engineering, (2022), vol. 44, no. 6, p. 227.

[8]. K. Sakkaravarthi, P. B. A. Reddy, and I. Sakthi, “Entropy optimization in Casson tetra-hybrid nanofluid flow over a rotating disk with nonlinear thermal radiation: A Levenberg–Marquardt neural network approach,” Journal of Computational Design and Engineering, (2024), vol. 11, no. 5, pp. 333–354.

[9]. M. Toufik and A. Atangana, “New numerical approximation of fractional derivative with non-local and non-singular kernel:

Application to chaotic models," The European Physical Journal Plus, (2017), vol. 132, pp. 1–16.

[10]. K. Hattaf, Z. Hajhouji, M. Ait Ichou, and N. Yousfi, "A numerical method for fractional differential equations with new generalized Hattaf fractional derivative," Mathematical Problems in Engineering, (2022), vol. 2022, p. 3358071.

[11]. N. Almutairi and S. Saber, "Chaos control and numerical solution of time-varying fractional Newton–Leipnik system using fractional Atangana–Baleanu derivatives," AIMS Mathematics, (2023), vol. 8, no. 11, pp. 25863–25887.

[12]. D. Baleanu, S. Qureshi, A. Yusuf, A. Soomro, and M. S. Osman, "Bi-modal COVID-19 transmission with Caputo fractional derivative using statistical epidemic cases," Partial Differential Equations in Applied Mathematics, (2024), vol. 10, p. 100732.

[13]. M. Dehghan, "Numerical solution of the three-dimensional parabolic equation with an integral condition," Numerical Methods for Partial Differential Equations, (2002), vol. 18, no. 2, pp. 193–202.

[14]. K. I. Falade, I. G. Baoku, and M. A. Lawan, "Numerical solution of a three-dimensional heat equation by homotopy perturbation algorithm," Unilag Journal of Mathematics and Applications, (2021), vol. 1, no. 1, pp. 1–19.

[15]. D. Jain, A. Bhargava, and S. Gupta, "A new approach to population growth model involving a logistic differential equation of fractional order," Critical Reviews in Biomedical Engineering, (2025), vol. 53.

[16]. S. Gupta, P. K. Jain, and D. Jain, "Thermal transport of 3D magnetohydrodynamic Jeffrey ternary hybrid nanofluid over an exponentially stretching surface along convective boundaries," Thermal Advances, (2025), p. 100047.

[17]. S. Gupta, D. Khandelwal, P. K. Jain, and V. K. Singhal, "EMHD flow of MoS_2/GO three-dimensional hybrid nanofluid with gyrotactic microorganisms and Cattaneo–Christov heat model," International Journal of Ambient Energy, (2025), vol. 46, no. 1, p. 2556771.