Ambiguity Function Analysis of LFM and Stepped LFM Waveforms

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Abstract- In this study, we investigate and compare the ambiguity functions (AFs) of LFM and Stepped LFM signals when the bandwidth, pulse duration, and operating conditions are identical. By matching these parameters, the comparison in this study enables direct separation between waveform effects and system mismatches/limitations. This observation emphasises the differences in the range resolution, Doppler tolerance, and sidelobe properties of these two signals. The results indicate that although both waveforms can achieve high-resolution radar imaging capabilities, each provides different design tradeoffs between the implementation effort and performance. In contrast to analogous works that treat both approaches independently, our evaluation of the two chosen methods allows us to obtain a practical view of waveform selection in high-resolution SAR.

Keywords– Ambiguity Function, Linear Frequency Modulation (LFM), Stepped LFM, Zero delay, Zero Doppler.

1. INTRODUCTION

Synthetic Aperture Radar (SAR) is an active remote sensing technology widely employed for high-resolution imaging of the Earth's surface and imaging objects under a variety of conditions[1], [2]. The range resolution in SAR largely depends on the bandwidth of the transmitted signal and has an inverse relationship; thus, ultra-wideband waveforms provide finer resolutions [3], [4].
Linear Frequency Modulation (LFM) pulses have long been employed in SAR because of their pulse compression capability and broad bandwidth However, wideband LFM signals, due to their large instantaneous bandwidth can present hardware challenges: their large instantaneous frequency may exceed the sampling capabilities of analog-to-digital converters (ADCs), complicating real-time sampling and processing [5]. Therefore, the resolution capability of a traditional LFM waveform is limited by the sampling constraints of the ADC.
Stepped LFM waveforms can be used to address these issues [6], [7], [8]. In stepped LFM By transmitting a sequence of stepped frequency pulses, Stepped LFM achieves a highly effective bandwidth by combining these pulses while reducing the instantaneous bandwidth requirement, easing ADC sampling, and enabling practical implementation in high-resolution SAR systems [9], [10].
In radar analysis, the ambiguity function describes how a signal behaves under time delays and Doppler shifts, making it crucial for understanding the resolution and sidelobe effects [5], [11]. Their study highlighted how the precise evaluation of azimuth and range ambiguities is essential for high-resolution SAR system design [12].
Although both LFM and Stepped LFM have been studied extensively, most prior analyses treat them independently, making direct comparison difficult. Previous works have examined Stepped LFM waveforms for SAR imaging and AF characteristics [7], [8], [9], while theoretical foundations for ambiguity analysis of diverse waveforms have been discussed in detail [6], [12]. Methods for improving the range resolution and sidelobe suppression using step-frequency trains have also been proposed [10], [13], and advanced waveform design approaches leveraging multi-parameter modulation have been explored in sonar and radar applications [14]. More recently, gradient-based optimisation of radar ambiguity functions has been introduced to enhance waveform performance [15], and integrated LFM-based frequency-hopping designs have demonstrated improved Doppler and range resolution in high-altitude platforms [16].
This study addresses this gap by presenting a side-by-side comparison of LFM and Stepped LFM using the same AF framework. Three-dimensional ambiguous surfaces, together with zero-delay and zero-Doppler cuts, are used to study key performance measures such as resolution, sidelobe levels, and hardware implications. Unlike many earlier studies that analysed both waveforms separately and with different parameter settings, our comparison was carried out under identical bandwidth, sampling rate, and pulse duration. This allows a fair and clear evaluation of the differences between the two waveforms and helps in understanding how waveform design choices influence radar performance under practical conditions.
The remainder of this paper is organised as follows: Section 2 introduces the mathematical background and AF expressions, Section 3 outlines the simulation setup, Section 4 presents the results, and Section 5 concludes with the key findings and future research directions.

2. MATHEMATICAL FOUNDATIONS AND ANALYTICAL EXPRESSIONS FOR LFM AND STEPPED LFM AMBIGUITY FUNCTIONS

This section presents the mathematical framework for comparing the ambiguity functions of LFM and Stepped LFM signals. We outline their signal models and derive analytical expressions of their ambiguity functions to evaluate the range resolution, Doppler tolerance, and sidelobe behaviour, forming the basis for the simulations in subsequent sections.

2.1 Linear Frequency Modulation (LFM)

The time-domain representation of the LFM signal with center frequency (f_c) can be expressed as

$$s(t) = rect\left(\frac{t}{T}\right) \cdot exp\{j2\pi(f_c t + \pi K t^2\} \quad (1)$$

where (t) is the time axis of the transmitted pulse, T is the pulse width, and the slope K is called the chirp rate. The phase term $\phi(\pi K t^2)$ indicates quadratic modulation owing to the linear frequency sweep, and $rect\left(\frac{t}{T}\right)$ is a rectangular window function that limits the signal duration to $\left[-\frac{T}{2}, \frac{T}{2}\right]$.

The **instantaneous bandwidth** is $B = |K| \cdot T$,

and the corresponding **range resolution** is $\Delta R = \frac{c}{2B}$. where c is the speed of light. with. This implies that a larger bandwidth leads to a finer range resolution. [1].

2.2 Stepped Linear Frequency Modulation (Stepped LFM)

In stepped LFM, each pulse is shifted by a frequency increment by transmitting a sequence of pulses, where each pulse is shifted by a frequency increment Δf. The signal is:

$$s_n(t) = rect\left(\frac{t}{T}\right) \cdot \exp\{j2\pi(f_c + n\Delta f)\, t + j\pi K t^2\} \quad , \quad n = 1,2, \ldots .. N$$

where f_c is the carrier frequency of the radar and Δf is the frequency step size [6].

Although each subpulse has the same bandwidth as a single LFM pulse, the effective bandwidth after coherent processing of all N pulses becomes, $B_{eff} = N \cdot B$ and the range resolution improves accordingly: $\Delta R = \frac{c}{2B_{eff}} = \frac{c}{2N\Delta f}$. Thus, stepped LFM enhances the resolution compared to a conventional single LFM pulse while reducing instantaneous bandwidth requirements [4].

2.3 Ambiguity Function

According to [5] [11], the ambiguity function (AF) quantifies how a radar waveform responds when it is simultaneously shifted in time (delay τ) and frequency (Doppler shift f_d). For a transmitted signal $s(t)$, the AF is defined as

$$|\chi(\tau, f_d)| = \left|\int_{-\infty}^{\infty} s(t) s^*(t+\tau) \exp\,(j2\pi f_d t) dt\right|$$

where s is the complex envelope of the transmitted signal, and $s^*(t+\tau)$ is a complex conjugate of the signal shifted by τ. The term $\exp\,(j2\pi f_d t)$ represents the modulation owing to the Doppler shift.

This function provides a two-dimensional characterisation of the waveform in both **the delay** and **Doppler** domains, making it a fundamental tool for evaluating the resolution, sidelobe levels, and range–Doppler coupling.

2.3.1 Ambiguity function of LFM

For an LFM pulse, the AF expression is

$$|\chi(\tau, f_d)| = \left| \left(1 - \frac{|\tau|}{T}\right) \frac{\sin\left[\pi T\left(f_d \mp B\left(\frac{\tau}{T}\right)\right)\left(1 - \frac{|\tau|}{T}\right)\right]}{\pi T\left(f_d \mp B\left(\frac{\tau}{T}\right)\right)\left(1 - \frac{|\tau|}{T}\right)} \right|$$

$$|\tau| \leq T \text{ , zero elsewhere}$$

This illustrates the range–Doppler coupling inherent in chirp signals, where the delay and frequency are not independent.

2.3.2 Ambiguity Function of Stepped LFM

According to [5], for a N-pulse stepped-frequency train, AF is derived by combining a single-pulse AF with a coherent pulse sequence. The result is:

$$|\chi(\tau, f_d)| = \left|\left(1 - \frac{|\tau|}{T}\right) \frac{\sin \alpha}{\alpha}\right| \left|\frac{\sin N\, \pi f_d T_r}{N \sin \pi f_d T_r}\right| ,$$

$$|\tau| \leq T$$

where $\alpha = \pi T\,\left(f_d \mp B\frac{\tau}{T}\right)\left(1 - \frac{\tau}{T}\right)$.

This approach considers the contributions of all sub-pulse pairs, along with their respective time delays and frequency shifts. It provides a way to analyse and compare the behaviour of LFM and Stepped LFM waveforms. By examining these expressions and their corresponding ambiguity functions, we can understand important waveform features, such as range resolution, Doppler tolerance, and sidelobe behaviour. Based on this analysis, the waveforms were then simulated to observe how they perform under different operating conditions.

3. Simulation and Methodology

This section describes the simulations carried out to obtain the ambiguity functions of the LFM and Stepped LFM waveforms. All computations were performed in MATLAB, which was used to generate the signals and visualise their ambiguity functions. The parameters were selected to provide a fair comparison between the two waveforms, while keeping the models sufficiently simple for clear interpretation.

The parameters used to generate the LFM pulse are tabulated in Table 1 as

Table 1: Parameters used to generate LFM pulse

Parameters	Values(Units)
Sampling Rate	3 (MHz)
Pulse Repetition frequency (PRF)	10 (KHz)
Pulse Repetition Interval (PRI)	100 (microsecond)
Number of Pulses (N)	1
Pulse Width (T_p)	10 (microseconds)

In contrast, the Stepped LFM signal was constructed using five subpulses, each maintaining a constant frequency that progressively increased linearly from one subpulse to the subsequent one.
The parameters used to generate the stepped LFM pulse train are presented in Table 2.

Table 2: Parameters used to generate Stepped LFM pulse train

Parameters	Values(Units)
Sampling Rate	3 (MHz)
Pulse Repetition frequency (PRF)	10 (KHz)
Number of Pulses (N)	5
Pulse Width (T_p)	10 (microseconds)
Frequency step size	300 (KHz)
Number of frequency steps	5
Bandwidth (Pulse)	1 (Mhz)

Using the values specified for the LFM and Stepped LFM pulse trains, we generated and obtained the waveform characteristics of both types of signals.

3.1 3D Ambiguity Function Plot for LFM and Stepped LFM Signals

The 3D ambiguity function for a radar signal is a 3D representation of the waveform response in terms of the range, Doppler, and amplitude.
For LFM waveforms, as shown in Figure 1, the ambiguity function typically exhibits a diagonal ridge in the range-Doppler plane, indicating a coupling between the range and Doppler measurements.
In contrast, Stepped LFM waveforms (Figure 2) produce a more complex AF with multiple peaks, potentially offering improved range-Doppler resolution, but at the cost of increased ambiguity in certain regions of the range-Doppler space.

3.2 The Zero-Delay Cut (Doppler Profile)

The zero-delay cut was obtained by setting the time delay to zero in the AF. For LFM signals figure. 3, it features a narrow, symmetrical main lobe with low and smooth sidelobes, indicating good Doppler resolution and consistent signal clarity.

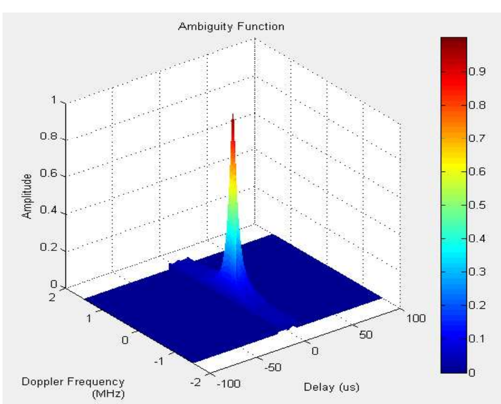

Figure 1: AF Plot for LFM pulse

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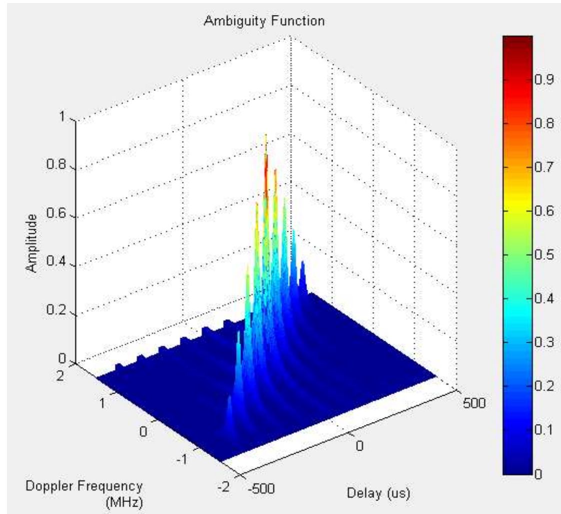

Figure 2: AF Plot for Stepped LFM pulse train

For comparison, a stepped LFM zero-delay cut figure is shown. Four has a wider main lobe, higher and more irregular sidelobes, and some asymmetry, which can reduce the Doppler resolution and affect the signal uniformity.
Finally, the stepped LFM exhibits more abrupt changes in the sidelobe levels, which can affect the overall signal performance.

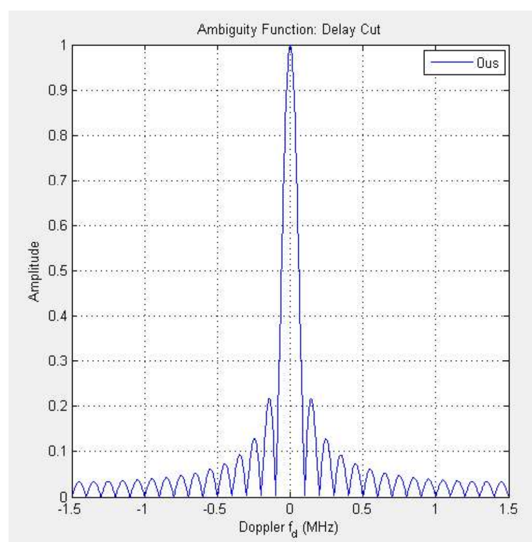

Figure 3: Zero-Delay Cut for LFM pulse

3.3 The Zero-Doppler Cut (Autocorrelation Function)

The zero Doppler cut is obtained by setting the Doppler frequency to zero in the ambiguity function equation. The zero-Doppler cut for a Linear Frequency Modulated (LFM) signal in Figure 5 typically shows a narrow main lobe with relatively low sidelobes, indicating good range resolution.

For the Stepped LFM signal, the zero-Doppler cut in Figure 6 also shows a narrow main lobe, but the sidelobe pattern is more irregular because of the frequency-stepped structure.

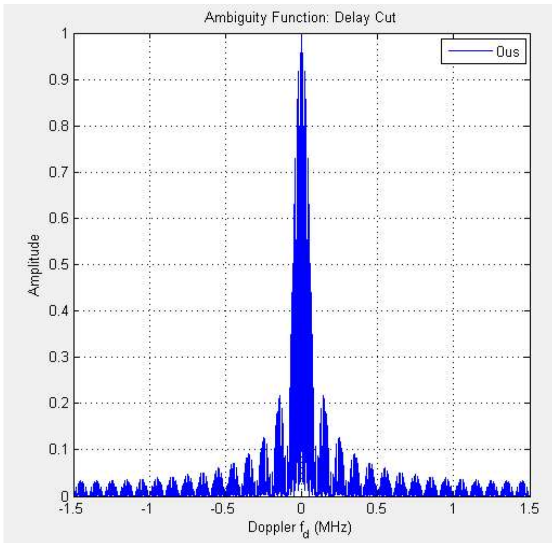

Figure 4: Zero-Delay Cut for Stepped LFM pulse train

When compared directly, the LFM signal tends to produce smoother sidelobes and a slightly better range resolution near the target. Stepped LFM, on the other hand, is more effective at reducing range–Doppler ambiguities, although this benefit can result in a minor loss of resolution at short ranges.

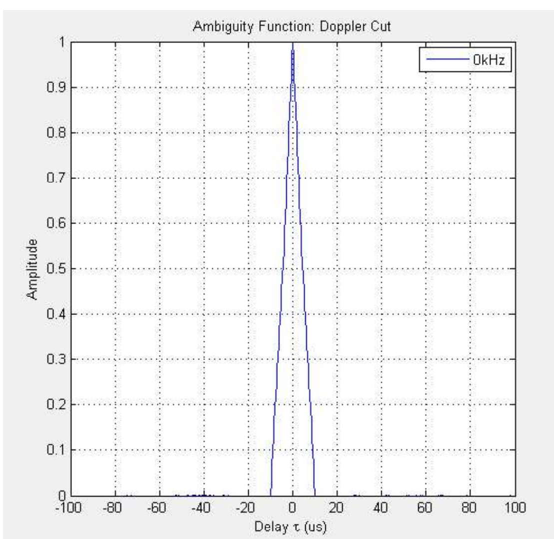

Figure 5: Zero-Doppler Cut for LFM pulse

Therefore, the choice between the two signals depends on the design requirements of the radar system and the operating environment. In the next section, we analyse these results in more detail and compare them with the experimental findings.

4. RESULT AND DISCUSSION

The ambiguity functions of both the LFM and Stepped LFM signals were simulated using the parameters defined in Tables 1 and 2, and the resulting waveform characteristics are summarised in Tables 3 and 4. These values highlight the differences in the achievable range resolution and time–bandwidth product, which are clearly reflected in the simulation outputs.

Three-dimensional AF revealed notable distinctions between the two waveforms. In our MATLAB simulations, the LFM waveform produced a diagonal ridge pattern in the range–Doppler domain, consistent with its well-known coupling behaviour.

In comparison, the stepped LFM waveform exhibits multiple peaks and a more complex structure.

While this complexity introduces additional sidelobe regions, it also achieves finer resolution along both the range and Doppler axes owing to the larger effective bandwidth.

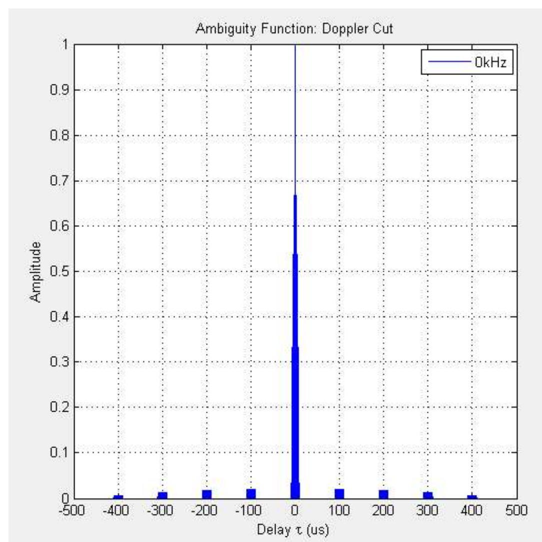

Figure 6: Zero-Doppler Cut for Stepped LFM pulse train

Table 3: Waveform Characterstics obtained for Single LFM pulse

Parameters	Formula used	Values (Units)
Range Resolution (ΔR)	$\frac{c}{2B}$	1.5 (Km)
Time Bandwidth Product (TBP)	$T_p \cdot BW_{pulse}$	$1 * 10^9$
Duty Cycle	$\frac{T_p}{PRI} * 100$	10 %
Pulse repetation Interval (PRI)	$1/_{PRF}$	100 (microsecond)
Bandwidth (pulse/total)	$\frac{1}{T_p}$	1 (Mhz)

An examination of the Doppler profiles further highlights these differences. The LFM waveform has a narrow main lobe with smooth, gradually decaying sidelobes, reflecting relatively good Doppler resolution and reduced interference. The stepped LFM signal shows a broader main lobe, slightly lowering Doppler resolution, and its sidelobes are higher and less uniform, with occasional asymmetry.

Although this can introduce interference, the trade-off allows the stepped waveform to achieve a significantly higher range resolution.

The range profiles provide complementary insights. The LFM pulse produces a narrow mainlobe with relatively smooth sidelobes, consistent with the 1.5 km resolution shown in Table 3.

The stepped LFM waveform also has a sharp main lobe, but its sidelobes are less regular owing to the stepped frequency structure. The larger effective bandwidth of 5 MHz improves the resolution to approximately 100 m (Table 4), providing better target discrimination, even though the irregular sidelobes could complicate detection under some conditions.

Overall, these results demonstrate the complementary strengths of the two waveforms. A single LFM pulse offers simpler processing and more stable sidelobe behaviour, but at the cost of a coarser range resolution.

Stepped LFM improves resolution and reduces range–Doppler coupling while introducing higher sidelobes and a more complex ambiguity pattern.

Table 4: Waveform Characterstics obtained for Stepped LFM pulse train for N = 5 pulses

Parameters	Formula used	Values (units)
Range Resolution (ΔR)	$\frac{c}{2B_{total}}$	100 (meter)
Time Bandwidth Product (TBP)	$T_p \cdot B_{total}$	$75 * 10^9$
Duty Cycle	$\frac{T_p}{PRI} * 100$	10 %
Pulse Repetation Interval (PRI)	$1/_{PRF}$	100 (microseconds)
Bandwidth (Total)	$N \cdot B_{pulse}$	5 (MHz)

The choice between the two depends on the specific radar application: LFM is suitable for systems prioritising implementation simplicity, whereas stepped LFM is preferred when fine resolution and reduced ambiguities are essential.

4.1 Comparison with Earlier Work

The AF depends strongly on parameters such as the bandwidth, pulse width, chirp rate, sampling rate, and number of frequency steps. Because earlier studies used different waveform settings, direct numerical comparison of AF plots is not meaningful. Therefore, a qualitative comparison is more suitable.

The LFM ambiguity pattern obtained in our simulations shows the same ridge-type structure and smooth sidelobes as those described in [5] and [11]. The stepped-LFM results also match the typical behaviour reported in stepped-frequency studies, including the multi-peak ambiguity pattern and reduced delay–Doppler coupling, as noted in [2,5] and [10,11].

By analysing both waveforms under identical bandwidth, pulse width, and sampling conditions, the present work agrees with the general trends seen in earlier literature while also providing a clearer, side-by-side comparison. This helps in understanding the trade-offs between smoother sidelobes in LFM and the higher effective bandwidth achieved by the stepped-LFM.

5. CONCLUSION

A comparative analysis of the LFM and stepped LFM waveforms shows that the individual characteristics of both signals can be exploited for SAR imaging. LFM pulses remain attractive because of their simple implementation and stable sidelobe behaviour, whereas stepped LFM pulses achieve finer range resolution and reduced range–Doppler coupling, albeit at the cost of greater signal complexity. Therefore, the choice between the two depends on the priorities of a particular system design.

There appear to be several promising directions for the future. One line of work is to examine how the ambiguity function is not only used as an analysis tool but can also be used as a guide for designing waveforms in more advanced radar architectures, such as MIMO systems, where delay–Doppler properties directly affect spatial diversity and target separability.

There is also growing interest in integrating ambiguity function design with optimisation methods [15]. showed that differentiable ambiguity functions as a gradient-based approach, which can be combined with artificial intelligence and machine learning techniques to adapt signals in real time.

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