An Algorithm for Interval-valued Fuzzy Fractional Transportation problem

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Abstract: **In this paper an approach is presented to solve Interval-valued fuzzy fractional transportation problem. It is proved that we can convert interval-valued fractional transportation problem under fuzzy environment to a linear fuzzy transportation problem, then Simplex method is used to get the optimum value of the given Interval-valued fuzzy fractional transportation problem (IVFFTP).**

1. INTRODUCTION

A fractional programming problem optimizes ratios of functions such as profit/cost, inventory/sales etc. In these models rates needs to be optimized. Here, we consider a special case of Transportation problem where the fractional objective function is the unit profit cost and the unit cost for transportation. In the real world problems, some parameters of the problem may not be known certainly due to uncontrollable factors. Hence, here we consider IVFFTP. The transportation problem was initially developed by Hitchcock [1] in 1941. Fuzzy members introduced by Zadeh [2]. Stancu, Minasian and Tigan [3,4] gave an algorithm for interval-valued optimization problem. Hsien-Chung Wu [5,6] proved and derived the KKT optimality conditions for an optimization problem with interval valued objective function. Effati and Pakdaman [7] solved the Interval-Valued Linear Fractional Programming Problem. Liu[8] solved Fractional Transportation problem with Fuzzy parameters. Here IVFFTP is solved by converting it to an optimization problem with Interval-Valued fuzzy objective function to get the optimal solution, which is illustrated with an example.

2. PRELIMINARIES

Let I be the set of all closed and bounded intervals in R. Suppose $A, B \in R$, then we write $A = [a^L, a^U]$ and also $B = [b^L, b^U]$, therefore on I we have the following operations:

(1) $A + B = \{a + b \mid a \in A, b \in B\} = [a^L + b^L, a^U + b^U] \in I;$

(2) $-A = \{-a \mid a \in A\} = [-a^U, -a^L] \in I;$

(3) If k is a real number, then we have $kA = \{ka \mid a \in A\} = [ka^U, ka^L] \in I;$

(4) $A - B = A + (-B) = [a^L - b^U, a^U - b^L]$

If A and B are positive intervals $i.e. 0 \le a^L \le a^U$ and $0 \le b^L \le b^U$ then we have: $AB = [a^L b^L, a^U b^U]$ and if $0 \le a^L \le a^U$ and $b^L < 0 < b^U$ then we have: $AB = [a^U b^L, a^L b^U]$

Theorem 2.1 Let $A = [a^L, a^U]$ and $B = [b^L, b^U]$ be two nonempty bounded real intervals and of $0 \notin [b^L, b^U]$, (see [9]) we have: $\frac{A}{B} = [a^L, a^U]\left[\frac{1}{b^L}, \frac{1}{b^U}\right]$

Theorem 2.2 If A and B are nonempty, bounded, real intervals, then $A+B$, $A-B$ and AB and if B does not contain zero, then A/B is also a nonempty, bounded, real interval as well, see([10]).

2.1 Arithmetic operations of Fuzzy numbers

Let $\tilde{a}=(a_1, a_2, a_3, a_4)$ and $\tilde{b}=(b_1, b_2, b_3, b_4)$ and be two trapezoidal fuzzy numbers, the arithmetic operators on these numbers are as follows

Addition: $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction: $\tilde{a} - \tilde{b} = (a_1 - b_2, a_2 - b_1, a_3 - b_4, a_4 - b_3)$

Scalar Multiplication:

$k\tilde{a} = (ka_1, ka_2, ka_3, ka_4), k > 0$

$k\tilde{a} = (ka_2, ka_1, -ka_4, -ka_3), k < 0$

Ranking Function: We define a ranking function $R : F(R) \to R,$ which maps each fuzzy number into the real line.

$$R(\tilde{a}) = \tilde{a}_1 + \tilde{a}_2 + \frac{\tilde{a}_4 - \tilde{a}_3}{2}$$

3. MATHEMATICAL FORMULATION OF IVFFTP

Let there are m origins, i^{th} origin possessing a_i units of a certain product, whereas there are n destinations with destination j requiring b_j units. The fractional objective function is the unit profit cost $\tilde{c}$ and the unit cost $\tilde{d}$ for transportation. Let x_{ij} be the number of units to be transported from i^{th} origin to j^{th} destination. It is also assumed that total availabilities $\sum a_i$, satisfy the total requirements $\sum b_j$ i.e., $\sum a_i = \sum b_j$. The mathematical formulation of a IVFFTP is

$$\max z = \frac{\tilde{c}\, x_{ij}}{\tilde{d}\, x_{ij}}$$

$$\text{s.t.} \sum_{i=1}^{m} x_{ij} = a_i, i = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j = 1,2,...,m$$

$$x_{ij} \geq 0 \qquad \ldots(1)$$

Suppose that $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)$ and $\tilde{d} = (\tilde{d}_1, \tilde{d}_2, ..., \tilde{d}_n)$

We denote $\tilde{c}_j^L$ and $\tilde{d}_j^L$ the lower bounds of the intervals i.e. $\tilde{c}^L = (\tilde{c}_1^L, \tilde{c}_2^L, ..., \tilde{c}_n^L)$ and also $\tilde{d}^L = (\tilde{d}_1^L, \tilde{d}_2^L, ..., \tilde{d}_n^L)$, where $\tilde{c}_j^L$ and $\tilde{d}_j^L$ are real scalars for $j = 1,2,...,n$.

Similarly, we denote $\tilde{c}_j^U$ and $\tilde{d}_j^U$ the upper bounds of the intervals i.e. $\tilde{c}^U = (\tilde{c}_1^U, \tilde{c}_2^U, ..., \tilde{c}_n^U)$ and also $\tilde{d}^U = (\tilde{d}_1^U, \tilde{d}_2^U, ..., \tilde{d}_n^U)$, where $\tilde{c}_j^U$ and $\tilde{d}_j^U$ are real scalars for $j = 1,2,...,n$.

We can rewrite (1) as follows

$$\max z = \frac{\tilde{p}(x)}{\tilde{q}(x)}$$

$$\text{s.t.} \sum_{i=1}^{m} x_{ij} = a_i, i = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j = 1,2,...,m$$

$$x_{ij} \geq 0 \qquad \ldots(2)$$

Where $\tilde{p}(x)$ and $\tilde{q}(x)$ are interval-valued linear functions as,

$\tilde{p}(x) = [\tilde{p}^L(x), \tilde{p}^U(x)] = [\tilde{c}^L x, \tilde{c}^U x]$ and

$\tilde{q}(x) = [\tilde{q}^L(x), \tilde{q}^U(x)] = [\tilde{d}^L x, \tilde{d}^U x]$

Therefore, from (2) we have $\max z = \dfrac{[\tilde{c}^L x, \tilde{c}^U x]}{[\tilde{d}^L x, \tilde{d}^U x]}$

$$s.t. \sum_{i=1}^{m} x_{ij} = a_i, i = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j = 1,2,...,m$$

$$x_{ij} \geq 0 \qquad \ldots(3)$$

Now, we can consider another kind of possible linear fractional programming problem as follows:

$$\max z = \left[\tilde{f}^L(x), \tilde{f}^U(x)\right]$$

$$\text{s.t.} \sum_{i=1}^{m} x_{ij} = a_i, i = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j = 1,2,...,m$$

$$x_{ij} \geq 0 \qquad \ldots(4)$$

Where $\tilde{f}^L$ and $\tilde{f}^U$ are linear fractional functions. Now, equation (3) under some assumptions can be converted to (4). The objective function in (3) is a quotient of two interval valued functions ($p(x)$ and $q(x)$). To convert (3) into (4), we suppose that, $0 < \tilde{q}^L(x) \leq \tilde{q}^U(x)$ for each feasible point x.

Therefore, we have $0 < \tilde{p}^L(x) \leq \tilde{p}^U(x)$ or $\tilde{q}^L(x) \leq \tilde{q}^U(x) < 0$ for each feasible point x. The denominator doesn't contain zero, we can rewrite the objective function in (3) using Theorem 2.1, as

$$z = \left[\tilde{c}^L x, \tilde{c}^U x\right]\left[\frac{1}{\tilde{d}^U x}, \frac{1}{\tilde{d}^L x}\right]$$

Now following cases are possible :

Case (1) : when $0 < \tilde{q}^L(x) \leq \tilde{q}^U(x)$ and $0 < \tilde{p}^L(x) \leq \tilde{p}^U(x)$ then we have

$$\max z = \frac{[\tilde{c}^L x, \tilde{c}^U x]}{[\tilde{d}^U x, \tilde{d}^L x]}$$

Case (2) : when $0 < \tilde{q}^L(x) \leq \tilde{q}^U(x)$ and $\tilde{p}^L(x) < 0 < \tilde{p}^U(x)$ then we have

$$\max z = \frac{[\tilde{c}^{L} x, \tilde{c}^{U} x]}{[\tilde{d}^{L} x, \tilde{d}^{L} x]}$$

Case (3) : when $\tilde{q}^{L}(x) \le \tilde{q}^{U}(x) < 0$ and $0 \le \tilde{p}^{L}(x) \le \tilde{p}^{U}(x)$ then we have

$$\max z = \frac{[\tilde{c}^{U} x, \tilde{c}^{L} x]}{[\tilde{d}^{U} x, \tilde{d}^{L} x]}$$

Case (4) : when $\tilde{q}^{L}(x) \le \tilde{q}^{U}(x) < 0$ and $\tilde{p}^{L}(x) < 0 < \tilde{p}^{U}(x)$ then we have

$$\max z = \frac{[\tilde{c}^{U} x, \tilde{c}^{L} x]}{[\tilde{d}^{U} x, \tilde{d}^{U} x]}$$

4. PROPOSED ALGORITHM

Consider an Interval-valued Fuzzy fractional Transportation problem:

$$\max z = \frac{\tilde{c}\, x_{ij}}{\tilde{d}\, x_{ij}}$$

$$\text{s.t. } \sum_{i=1}^{m} x_{ij} = a_i, i = 1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = b_j, j = 1,2,...,m$$

$$x_{ij} \ge 0$$

$\max z =$

$$\left[\frac{\begin{array}{l}((3,6,3,5)x_{11} + (4,5,4,4)x_{12} + (2,4,3,5)x_{21} + (4,6,2,4)x_{22}),\\ ((6,7,6,7)x_{11} + (8,6,9,8)x_{12} + (6,7,4,9)x_{21} + (9,7,7,8)x_{22})\end{array}}{\begin{array}{l}((3,1,2,2)x_{11} + (2,2,1,2)x_{12} + (1,1,2,2)x_{21} + (3,2,2,1)x_{22}),\\ ((4,4,4,4)x_{11} + (4,7,5,7)x_{12} + (5,5,8,5)x_{21} + (4,7,7,7)x_{22})\end{array}}\right]$$

$$\text{s.t. } x_{11} + x_{12} \le 40$$
$$x_{21} + x_{22} \le 20$$
$$x_{11} + x_{21} \le 30$$
$$x_{12} + x_{22} \le 30$$
$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0$$

Here, we see that

$$\tilde{p}(x) = [\tilde{p}^{L}(x), \tilde{p}^{U}(x)] = [\tilde{c}^{L} x, \tilde{c}^{U} x]$$

In the given problem we have, $0 < \tilde{p}^{L}(x) \le \tilde{p}^{U}(x)$

$$\tilde{q}(x) = [\tilde{q}^{L}(x), \tilde{q}^{U}(x)] = [\tilde{d}^{L} x, \tilde{d}^{U} x]$$

and,

$$0 < \tilde{q}^{L}(x) \le \tilde{q}^{U}(x)$$

$\therefore$ By applying the case (1), the objective becomes

$\max z =$

$$\left[\frac{(3,6,3,5)x_{11} + (4,5,4,4)x_{12} + (2,4,3,5)x_{21} + (4,6,2,4)x_{22}}{(4,4,4,4)x_{11} + (4,7,5,7)x_{12} + (5,5,8,5)x_{21} + (4,7,7,5)x_{22}} + \frac{(6,7,6,7)x_{11} + (8,6,9,8)x_{12} + (6,7,4,9)x_{21} + (9,7,5,8)x_{22}}{(3,1,2,2)x_{11} + (2,2,1,2)x_{12} + (1,1,2,2)x_{21} + (3,2,2,1)x_{22}}\right]$$

$$\text{s.t. } x_{11} + x_{12} \le 40$$
$$x_{21} + x_{22} \le 20$$
$$x_{11} + x_{21} \le 30$$
$$x_{12} + x_{22} \le 30$$
$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0$$

Which can be written as

$$\max z = z_1 + z_2,$$

Where $z_1 =$

$$\left[\frac{(3,6,3,5)x_{11} + (4,5,4,4)x_{12} + (2,4,3,5)x_{21} + (4,6,2,4)x_{22}}{(4,4,4,4)x_{11} + (4,7,5,7)x_{12} + (5,5,8,5)x_{21} + (4,7,7,5)x_{22}}\right]$$

and $z_2 =$

$$\left[\frac{(6,7,6,7)x_{11}+(8,6,9,8)x_{12}+(6,7,4,9)x_{21}+(9,7,5,8)x_{22}}{(3,1,2,2)x_{11}+(2,2,1,2)x_{12}+(1,1,2,2)x_{21}+(3,2,2,1)x_{22}}\right]$$

therefore, we should maximize z_1 and z_2, where the numerator functions are to be maximized and the denominator functions are to be minimized.

$$\therefore \max z_1^* = (-1,2,-1,1)x_{11}+(-3,1,-3,-1)x_{12}+(-3,-1,-2,-3)x_{21}+(-3,2,-3,-3)x_{22}$$

$$\max z_2^* = (5,4,4,5)x_{11}+(6,4,7,7)x_{12}+(5,6,2,7)x_{21}+(7,4,4,6)x_{22}$$

Now the given problem can be written in the standard form:

$$\max z = (4,6,3,6)x_{11}+(3,5,4,6)x_{12}+(2,5,0,4)x_{21}+(4,6,1,3)x_{22}$$

$$s.t.\ x_{11}+x_{12}\le 40$$
$$x_{21}+x_{22}\le 20$$
$$x_{11}+x_{21}\le 30$$
$$x_{12}+x_{22}\le 30$$
$$x_{11},x_{12},x_{21},x_{22}\ge 0$$

Adding slack variables

$$\max z = (4,6,3,6)x_{11}+(3,5,4,6)x_{12}+(2,5,0,4)x_{21}+(4,6,1,3)x_{22}$$

$$s.t.\ x_{11}+x_{12}+s_1=40$$
$$x_{21}+x_{22}+s_2=20$$
$$x_{11}+x_{21}+s_3=30$$
$$x_{12}+x_{22}+s_4=30;\ x_{11},x_{12},x_{21},x_{22},s_1,s_2,s_3,s_4\ge 0$$

Applying Simplex method

		C_j	(4,6,3,6)	(3,5,4,6)	(2,5,0,4)	(4,6,1,3)	0	0	0	0
Basic Variables	C_B	X_B	x_{11}	x_{12}	x_{21}	x_{22}	s_1	s_2	s_3	s_4
s_1	0	40	1	1	0	0	1	0	0	0
s_2	0	20	0	0	1	1	0	1	0	0
s_3	0	30	1	0	1	0	0	0	1	0
s_4	0	30	0	1	0	1	0	0	0	1
			(-6,-4,6,3)	(-5,-3,6,4)	(-5,-2,4,0)	(-6,-4,3,1)	0	0	0	0
Ranking Function R			-11.5	-9	-9	-11	0	0	0	0

		C_j	(4,6,3,6)	(3,5,4,6)	(2,5,0,4)	(4,6,1,3)	0	0	0	0
Basic Variables	C_B	X_B	x_{11}	x_{12}	x_{21}	x_{22}	s_1	s_2	s_3	s_4
s_1	0	10	0	1	-1	0	1	0	-1	0
s_2	0	20	0	0	1	1	0	1	0	0
x_{11}	(4,6,3,6)	30	1	0	1	0	0	0	1	0
s_4	0	30	0	1	0	1	0	0	0	1
			(-2,2,-3,3)	(-5,-3,6,4)	(-1,4,-1,6)	(-6,-4,3,1)	0	0	(4,6,3,6)	0
Ranking Function R			3	-9	8.5	-11	0	0	11.5	0

The last iteration will be

		C_j	(4,6,3,6)	(3,5,4,6)	(2,5,0,4)	(4,6,1,3)	0	0	0	0
Basic Variables	C_B	X_B	x_{11}	x_{12}	x_{21}	x_{22}	s_1	s_2	s_3	s_4
x_{12}	(3,5,4,6)	10	0	1	-1	0	1	0	-1	0
x_{22}	(4,6,1,3)	20	0	0	1	1	0	1	0	0
x_{11}	(4,6,3,6)	30	1	0	1	0	0	0	1	0
s_4	0	0	0	0	0	0	-1	0	0	1
			(-2,2,-3,3)	(-2,2,-2,2)	(0,5,-4,3)	(-2,2,-2,2)	(3,5,4,6)	(4,6,1,3)	(-1,3,9,10)	0
Ranking Function R			3	2	8.5	2	9	11	2.5	0

Since all $R \ge 0$, an optimum solution is obtained as

$$x_{11}=30, x_{12}=10, x_{22}=20$$

5. CONCLUSION

In this paper interval-valued fuzzy fractional transportation problem has converted to fuzzy linear transportation programming problem, which is further solved by Simplex Method.

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